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Error Analysis Of Students In Implementing The Newton-Raphson And Bisection Methods On Nonlinear Equation Root Problems Based On Newman's Error

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Abstract

This study aims to describe students' errors in solving nonlinear equation root problems using the Bisection and Newton-Raphson methods based on Newman's Error Analysis (NEA). This study uses a qualitative approach with descriptive methods. The research subjects consisted of 33 students of the Mathematics Education Study Program who had taken the Numerical Analysis course. The research instrument was a Mid-Semester Exam question on the material of nonlinear equation roots covering the Bisection and Newton-Raphson methods. Data collection techniques were carried out through written tests and semi-structured interviews, then analyzed based on Newman's error categories, namely reading, comprehension, transformation, process skills, and encoding. The results showed that the most dominant errors made by students were in the process skills category. In the Bisection method, student errors mostly occurred in the process of evaluating functions and determining new intervals during iteration, while in the Newton-Raphson method, dominant errors occurred in determining function derivatives and substituting iteration values into the Newton-Raphson formula. In addition, it was found that some students experienced transformation errors due to the inability to connect mathematical concepts with algorithmic procedures of numerical methods. Interview results indicate that student errors are influenced by a lack of understanding of basic concepts, low accuracy in numerical operations, and a lack of habit of double checking calculation results. This research is expected to serve as a basis for designing Numerical Analysis learning strategies that emphasize students' conceptual understanding and procedural accuracy.

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Keyword: Error Analysis, Newman's Error Analysis, Bisection Method, Newton-Raphson Method.

INTRODUCTION

Problem solving ability is one of the core competencies in mathematics learning in higher education, especially in the context of science, technology and engineering. (Amalina & Vidákovich, 2023). This competency encompasses not only the ability to obtain correct answers but also involves higher-order thinking processes such as conceptual understanding, strategy selection, and reflection on the obtained solutions (Siller et al., 2025). Within this framework, solving nonlinear equations becomes an essential topic because many real-world problems lack closed analytical solutions, thus requiring a systematic and structured numerical approach (Hamayd et al., 2025)

Numerical methods such as the Newton-Raphson method and the Bisection method are classical techniques used to determine the roots of nonlinear equations (Hamayd et al., 2025). The Newton-Raphson method is known for its fast (quadratic) convergence rate, but it is very sensitive to the choice of initial values and requires a strong understanding of the concept of derivatives (Cuevas et al., 2024). In contrast, the Bisection method offers stability and guaranteed convergence based on the intermediate value theorem, albeit at a slower rate (Abdulwahhab & Ramadan, 2025). These differences in characteristics require students not only to understand algorithmic procedures but also to be able to exercise mathematical judgment in selecting a method that suits the problem structure.

However, various studies in mathematics education show that students often experience difficulties integrating conceptual understanding and procedural skills, particularly in algorithmic materials such as numerical methods (Hadley & Oyetunji, 2022). The errors that occur are not solely computational errors, but also reflect a failure to understand the mathematical meaning of the procedures used (Sánchez-Peña et al., 2023). In this context, error analysis is a relevant approach for in-depth exploration of students' cognitive processes in solving mathematical problems.

One framework used to analyze errors in mathematical problem-solving is Newman's Error Analysis. This framework classifies errors into five stages: reading, comprehension, transformation, process skills, and encoding (Doni & Prabawanto, 2025). The advantage of this approach lies in its ability to hierarchically identify the location of errors in students' thought processes, allowing researchers to distinguish between conceptual and procedural errors (Irianti et al., 2024).

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Several previous studies have applied Newman's framework to various mathematics learning contexts and shown that student errors tend to be concentrated in the transformation and process skills stages (Doni & Prabawanto, 2025). However, most of these studies focus on school mathematics or basic topics, such as algebra and arithmetic, with relatively limited attention to advanced mathematical contexts, particularly numerical analysis. Furthermore, research examining student errors in implementing numerical methods often emphasizes the accuracy of the results rather than analyzing the underlying cognitive processes.

Furthermore, studies comparatively analyzing the error profiles of students using two numerical methods with different characteristics, such as the derivative-based Newton-Raphson and the interval-based Bisection, are still rare. Yet, each method has its own specific potential for error. In the Newton-Raphson method, errors can arise from misconceptions about derivatives, errors in the iteration formulation, or an inability to interpret convergence. Meanwhile, in the Bisection method, errors are often related to a failure to understand the condition for changing the function's sign on the initial interval, as well as errors in the recursive iterative process

The limitations in this literature indicate a significant research gap: there are not many studies integrating Newman's framework-based error analysis with the specific context of numerical methods at the higher education level. Therefore, a study is needed that not only identifies the types of student errors but also links them to the epistemological characteristics of the methods used, thereby gaining a more comprehensive understanding of the sources of student learning difficulties.

Based on this background, this study aims to analyze the types and causes of student errors in solving nonlinear equation root problems using the Newton-Raphson and Bisection methods based on Newman's Error Analysis approach. Specifically, this study seeks to (1) identify the distribution of errors at each Newman stage, (2) compare error profiles between the two methods, and (3) uncover the factors influencing these errors.

Theoretically, this research is expected to enrich research in mathematics education, particularly related to error analysis in numerical analysis, a topic that remains relatively underexplored. Practically, the findings of this study are expected to be the basis for designing more effective learning interventions, such as strengthening conceptual connections, procedural scaffolding, and the use of error-based learning approaches to improve the quality of mathematics learning in higher education.

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METHOD

The purpose of this study was to determine student errors in solving nonlinear equation root problems using the Newton-Raphson method and the Bisection method based on Newman's Error Analysis. This study employed a qualitative approach with descriptive methods.

The subjects in this study were students in the Mathematics Education study program who had taken the Numerical Analysis course. Subjects were selected using a purposive sampling technique, taking into account students' academic abilities. This technique was used to obtain representative data regarding the variation in student errors in problem solving.

Data collection in this study utilized selected Midterm Exam questions focused on the topic of roots of nonlinear equations using the Bisection and Newton-Raphson methods. The questions used in this study are presented in Table 1.

Table 1. Midterm Exam Questions on the Topic of Roots of Nonlinear Equations

No	Soal
1	Given a nonlinear function: $f(x) = e^x - 3x$. a. Show that there are roots in the interval [0,1]! b. Find the roots using the bisection method!
2	Given a function $f(x) = x^3 - 2x - 5$. Find the roots of the equation of the function using the Newton-Raphson method with an initial guess $x_0 = 2$! Perform the calculation up to 3 iterations or $\lvert x_{r+1} - x_r \rvert < 0.001$!

These questions were chosen because they reveal various stages of students' mathematical thinking, particularly in understanding the concept of roots of nonlinear equations, selecting the appropriate method, and carrying out iterative procedures. Furthermore, the two methods used have different characteristics, allowing for a wider variety of errors.

Analysis of the students' answer sheets was conducted based on Newman's Error Analysis framework. This framework identifies five error categories: reading, comprehension, transformation, process skills, and encoding (Hidayah & Rejeki, 2022). Indicators for each category are presented in Table 2.

Table 2. Categories and Indicators of Newman's Error Analysis

No	Category	Indicator
1	*Reading (R): Error in simple*	Students misread symbols, notations, or

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	recognition of words and symbols	important information in questions.
2	*Comprehension (C): Error in understanding the meaning of a problem*	Students do not understand the meaning of the question or what is being asked
3	*Transformation (T): Error in transforming a word problem into an appropriate mathematical problem*	Students choose the wrong method or fail to form an iterative model.
4	*Process skills (PS): Error in performing mathematical procedures*	Students make mistakes in calculations or iteration steps
5	*Encoding (E): Error in representing the mathematical solution into acceptable written form*	The student did not write the final answer correctly or incompletely.

Data collection was conducted through a written test designed to measure students' ability to solve nonlinear equation root problems using the Newton-Raphson and Bisection methods. The questions were structured to highlight potential errors at various stages of the solution, from understanding the problem to writing the final answer. Interviews were also conducted to support the error analysis data on the students' answer sheets. These interviews aimed to identify the causes of errors made by students in solving the problems. The interview technique used was semi-structured, with questions based on a predetermined interview guide.

The validity of the data in this study was maintained through method triangulation, which involved comparing the written test results with the interviews. Thus, the results of this study are expected to provide a more accurate picture of students' errors in solving nonlinear equation root problems using the Newton-Raphson and Bisection methods.

RESULTS AND DISCUSSION

Based on the analysis of 33 student answers to the Bisection method questions, the most common errors were found to be transformation errors and process skills errors. Some students were able to correctly determine the initial interval and use the midpoint formula, but still made

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errors in determining the new interval based on the change in the function's sign. These errors caused the iteration process to be inaccurate, resulting in incorrect root approximations.

The results of the identification of student errors in solving Bisection method questions are presented in Table 3.

Table 3. Categories of Student Errors in Bisection Method Questions

No	Category Error	Indicator Error	Number of Students
1	*Reading (R)*	Misreading a symbol or interval	0
2	*Comprehension (C)*	Do not understand the requirements for changing function signs	6
3	*Transformation (T)*	Wrongly specifying new interval	9
4	*Process skills (PS)*	Iteration and function value calculation errors	13
5	*Encoding (E)*	Not writing the final conclusion properly	5

Table 3 shows that the most common errors made by students are process skills errors. Analysis of student answers using the Bisection method revealed that some students were able to correctly determine the initial interval and use the midpoint formula. However, errors were still found in determining the new interval based on the change in the function's sign. These errors cause the iteration process to drift away from the true root, resulting in inaccurate root approximations.

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Examples of student answers using the Bisection method are shown in Figure 1.

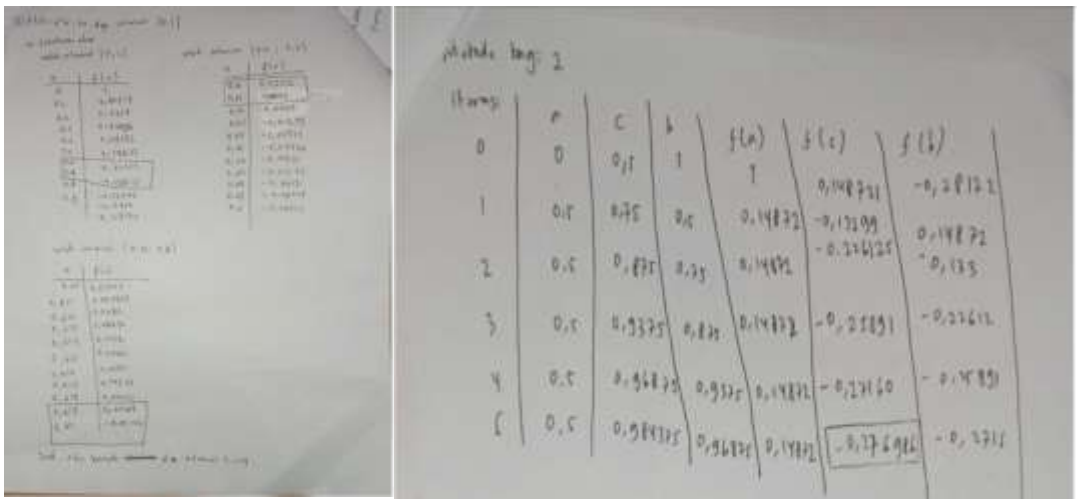


Figure 1. Errors in the transformation, process skill, and encoding stages in the Bisection Method

Figure 1 shows that the student was able to determine the function value at the initial interval and perform iterations using the midpoint formula. The student also systematically compiled an iteration table by writing down the values of a, b, and c, along with the function value at each iteration. However, the student made an error when determining a new interval after obtaining the function value at the midpoint. The selected iteration interval moved away from the actual root. As a result, the resulting root approximation did not match the expected solution.

Furthermore, in the root localization section, the student wrote that "the root is in the interval 0.619." This statement indicates that the student does not yet understand the concept of a root interval because the value 0.619 is an approximation point, not an interval. Therefore, the student's errors in the Bisection method include incorrectly determining the new interval after iteration (transformation error), moving the iteration away from the actual root (process skill error), and incorrectly writing the root point as the root interval (encoding error).

Furthermore, based on the analysis of 33 students' answers to the Newton-Raphson method, it was found that some students were able to determine the derivative of a function and use the iteration formula correctly. However, several students still made various errors in the iteration process, value substitution, use of the derivative of the function, and interpretation of error tolerance. These errors indicate that some students still have difficulty understanding the concept of numerical iteration conceptually and procedurally.

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The results of the identification of student errors in the Newton-Raphson method are presented in Table 4.

Table 4. Categories of Student Errors in Newton Raphson Method Questions

No	Category Error	Indicator Error	Number of Students
1	*Reading (R)*	Misreading a symbol or interval	0
2	*Comprehension (C)*	Not understanding the meaning of error tolerance and iteration convergence	5
3	*Transformation (T)*	Not understanding the meaning of error tolerance and iteration convergence	7
4	*Process skills (PS)*	Substitution errors of values and decimal arithmetic operations	12
5	*Encoding (E)*	Not writing the final conclusion properly	4

Table 4 shows that the most common errors made by students are process skills errors. These errors occur when substituting function values, calculating derivatives, and performing arithmetic operations during the iteration process. Furthermore, students were still unable to correctly apply the Newton-Raphson iteration formula, which falls into the category of transformation errors.

One example of a student's answer demonstrating an error in the Newton-Raphson method is shown in Figure 2.

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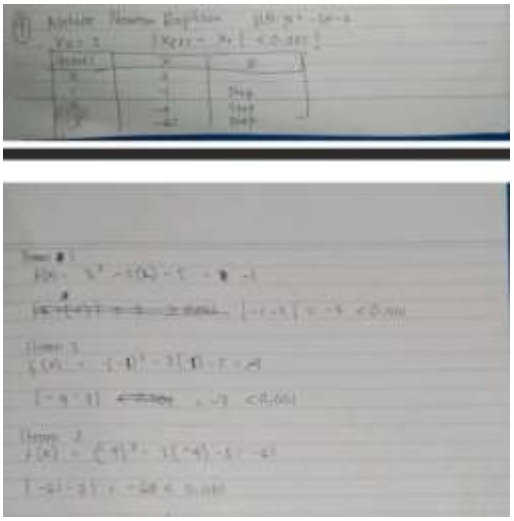


Figure 2. Errors in the transformation, process skill, and encoding stages of the Newton Raphson method

Based on Figure 2, the student only substituted function values without using the function derivative and the Newton-Raphson iteration formula. This error indicates that the student did not yet understand the relationship between functions, function derivatives, and the iteration process in the Newton-Raphson method. Furthermore, the student made an error in using absolute values when calculating errors. Absolute values should always be positive. A similar error occurred in the next iteration, causing the student to stop the iteration even though the actual error value was still far from the tolerance limit.

In other student answers, errors were found in function value substitution and inaccuracies in decimal arithmetic operations. Although the student correctly used the Newton-Raphson formula, some of the iteration results were imprecise due to numerical computation errors. Furthermore, some students failed to write down the final interpretation of the iteration results, resulting in incomplete solutions.

These errors fall into the comprehension error category because the student does not yet understand the meaning of error tolerance and iteration convergence. Mistakes in using the iteration formula are considered transformation errors, while errors in arithmetic operations and value substitution are considered process skills errors. At the end of the solution, an encoding error was also found when the student was unable to accurately write the conclusion of the iteration results.

Based on an analysis of student answers to the Bisection method, the most common errors were found in the process skills and encoding categories. Students generally understood

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the basic concepts of the Bisection method, but were still less thorough in determining new intervals, performing decimal operations, and writing the final conclusions of the iteration results. Some students were also inconsistent in writing interval notation and function values at each step of the solution. These findings indicate that errors occurred more frequently in the procedural stage than in understanding the basic concepts of the method. These results align with previous research that suggests process skills errors arise from a lack of thoroughness in the mathematical solution process (Sinaga et al., 2025).

In the Newton-Raphson method, student errors predominantly occurred in the transformation and process skills categories. Some students understood the Newton-Raphson formula but were unable to apply it correctly in the iteration process. Errors were seen in calculating function derivatives, substituting iteration values, and determining errors.

Some students even incorrectly terminated iterations due to misunderstandings about the concept of error tolerance. This indicates that students still struggle to connect mathematical concepts with systematic numerical procedures. This finding aligns with Newman's Error Analysis research, which states that transformation and process skills errors occur when students fail to apply solution procedures correctly (Ghefira & Jupri, 2025). The results of the study indicate that students generally understand the basic concepts of the Newton-Raphson method, but still experience difficulties in implementing iteration procedures accurately and consistently. This finding suggests that learning numerical methods needs to emphasize more on conceptual understanding related to the iteration process, the use of function derivatives, and error interpretation so that students not only memorize formulas but also understand the mathematical meaning of each solution step.

CONCLUSION

Based on the analysis of 33 students' answers to the Bisection and Newton-Raphson problem-solving methods, it was found that the most dominant student errors were in the process skills error and transformation error categories based on Newman's Error Analysis. In the Bisection method, students generally understood the basic concepts and were able to determine the initial interval, but still experienced errors in determining the new interval, performing decimal arithmetic operations, and writing the final conclusions of the iteration results. Meanwhile, in the Newton-Raphson method, errors often occurred in the use of iteration formulas, value substitution, calculation of function derivatives, and interpretation of error tolerance. The research findings indicate that some students have understood the basic concepts of numerical methods, but have not been able to implement iteration procedures

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correctly and consistently. Therefore, learning numerical methods needs to emphasize conceptual understanding, accuracy of numerical computation, and interpretation of the iteration process so that students are not only able to use formulas, but also understand the mathematical meaning of each step in the solution.

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