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Analysis Of Pre-Service Mathematics Teachers' Mathematical Reasoning Ability In Solving Solid Analytic Geometry Problems

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Abstract

This study aims to describe university students' mathematical reasoning ability in solving Solid Analytic Geometry (Geometri Analitik Ruang) problems. A descriptive qualitative approach was used, involving 32 students who took the Final Semester Examination (UAS) of the Solid Analytic Geometry course at Universitas Negeri Makassar. Students' responses to three problems covering the equation of a plane, the position of two lines in space, and the position of a sphere relative to a plane were scored using a 0-4 rubric and mapped onto four mathematical reasoning indicators (IPM): performing mathematical manipulation (IPM-2), constructing proof/giving reasons (IPM-3), drawing conclusions (IPM-4), and examining the validity of an argument (IPM-5). The results show that students' overall mathematical reasoning ability falls into the medium category (mean = 2.92), with 46.9% of students categorized as high, 43.8% as medium, and 9.4% as low. Among the four indicators, constructing proof/giving reasons (IPM-3) achieved the highest mean (3.13, high category), while performing mathematical manipulation (IPM-2) achieved the lowest mean (2.61, medium category). This finding indicates a gap between students' conceptual understanding and their procedural/computational skill, particularly in vector-algebra operations. Structured, gradual practice in mathematical manipulation is recommended, while maintaining instructional strategies that have proven effective in building students' argumentation skills

Keyword: mathematical reasoning; solid analytic geometry; reasoning indicators; qualitative descriptive; higher education.

INTRODUCTION

Mathematical reasoning is one of the fundamental competencies that constitutes a

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central goal of mathematics learning at all levels of education, including higher education (Herbert et al., 2015; NCTM, 2000). This competency encompasses the thinking processes involved in drawing conclusions, constructing arguments, and evaluating the validity of mathematical statements based on relevant facts, properties, or relationships (Asmaun & Baharuddin, 2024). In geometry learning, reasoning plays a particularly important role, as students are required not only to understand geometric objects visuo-spatially but also to manipulate them algebraically and to construct coherent deductive arguments (Clements & Battista, 1992).

Solid Analytic Geometry is a compulsory course in the Mathematics Education Study Program that requires an integration of spatial visualization ability and algebra-vector manipulation, for instance in determining the equation of a plane, the position of two lines in space, and the position of a sphere relative to a plane (Arwadi et al., 2024). The abstract and procedural nature of this material makes the course one that is relatively difficult for students to master, so that research on how students reason when solving problems in solid analytic geometry remains important to pursue further.

A number of previous studies have addressed the issue of students' mathematical ability in Analytic Geometry courses. Asmaun et al. (2025) described various types of student errors in solving Analytic Geometry problems based on Newman's theory, viewed from students' mathematical ability, and found that transformation errors and process-skill errors occurred predominantly among students with medium and low ability. In a related line of work, Asnawati & Dewi (2016) applied the MURDER technique with a metacognitive approach to improve students' mathematical reasoning in Analytic Geometry courses, indicating that students' mathematical reasoning in this subject still requires improvement through appropriate instructional interventions. Noto et al. (2016) further reported that students' mathematical representation and connection abilities in analytic geometry remain relatively low, particularly in the aspect of symbolic representation leading to problem solving. At a different level of geometry content, the present author previously examined students' geometric thinking levels based on Van Hiele's theory in the Basic Geometry course (Asmaun, 2025), which underscored the importance of measurably mapping students' thinking/reasoning achievement for the purpose of instructional improvement.

Studies on mathematical reasoning in solid-figure and other geometry topics have also been widely conducted at the secondary school level, such as Rohmah et al. (2020), who found variation in junior high school students' mathematical reasoning achievement on solid-figure topics when viewed from cognitive style, and Hariyanti & Khotimah (2022), who revealed

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differences in students' mathematical reasoning achievement on flat-sided solid-figure topics when viewed from gender. These studies, together with findings from the international context by Downton & Livy (2022) on students' geometric reasoning regarding prisms and Demiray (2026) on inductive reasoning strategies in non-Euclidean geometry, consistently show that mathematical reasoning ability in the geometry domain tends to vary across individuals and indicators, and frequently reveals an uneven achievement pattern between conceptual and procedural aspects. Similar findings in the broader context of mathematical reasoning have also been reported by Amalia (2017), Hasanah et al. (2019), Isnaeni et al. (2018) dan Putri et al. (2019).

Although these studies have provided a general picture of students' errors and ability levels, research specifically describing mathematics education students' mathematical reasoning achievement in the Solid Analytic Geometry course based on detailed mathematical reasoning indicators (IPM), using authentic Final Semester Examination (UAS) data, remains limited. This study seeks to fill that gap by describing students' mathematical reasoning achievement both overall and per indicator—namely performing mathematical manipulation, constructing proof/giving reasons for the validity of a solution, drawing conclusions from a statement, and examining the validity of an argument—following the reasoning indicators commonly used in mathematics education research (Herbert et al., 2015; NCTM, 2000). Based on the foregoing, this study aims to describe students' mathematical reasoning ability in solving Solid Analytic Geometry problems, both overall and in terms of each mathematical reasoning indicator, as a basis for improving instructional strategies in the course.

METHOD

This study employed a descriptive qualitative approach, chosen because the objective was to describe students' mathematical reasoning ability in solving Solid Analytic Geometry problems, viewed from the mathematical reasoning indicators. The research subjects were 32 students of Class C of the International Class Program (ICP) in the Mathematics Education Study Program who were enrolled in the Solid Analytic Geometry course, Department of Mathematics, Universitas Negeri Makassar, in the even semester of the 2025-2026 academic year.

The instrument used was the UAS test, consisting of 3 problems with a total of 13 statements/solution steps, as follows:

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UJIAN GEOMETRI ANALITIK RUANG

Pemahaman Konseptual: Analisis Syarat Perlu dan Cukup

Mata Kuliah	: Geometri Analitik Ruang	**Waktu**	: 120 menit
Semester / Kelas	:	**Sifat Ujian**	: Tutup Buku
Nama Mahasiswa	:	**NIM**	:

PETUNJUK PENGERJAAN: Bacalah setiap soal dengan teliti sebelum menjawab. Untuk soal Benar/Salah, tuliskan jawaban Anda (B atau S) dan berikan argumentasi singkat untuk setiap pernyataan. Untuk soal langkah penyelesaian, identifikasi langkah yang mengandung kesalahan dan berikan koreksinya secara lengkap. Kerjakan secara mandiri. Selamat bekerja.

SOAL 1 Persamaan Bidang Rata

Syarat perlu dan cukup agar dua bidang rata berimpit adalah vektor-vektor normalnya sejajar	B	S
Jika vektor $n = (A, B, C)$ adalah vektor normal bidang rata α, maka untuk setiap $k \in \mathbb{R}$ dengan $k \neq 0$, vektor (kA, kB, kC) juga merupakan vektor normal bidang α	B	S
Syarat perlu dan cukup agar persamaan $Ax + By + Cz + D = 0$ merepresentasikan suatu bidang rata adalah A, B, dan C tidak semuanya nol	B	S
Diberikan titik P_0 dengan vektor posisi r_0 serta dua vektor a dan b. Syarat perlu dan cukup agar $r = r_0 + \lambda a + \mu b$ merupakan persamaan vektoris suatu bidang rata adalah $a \times b \neq 0$	B	S

SOAL 2 Kedudukan Dua Garis Lurus dalam Ruang

Diberikan dua garis lurus:

$$g_1 : x = 1 + 2t,\ y = -1 + t,\ z = 3 - t$$
$$g_2 : x = 2 - s,\ y = 3 + 2s,\ z = 1 + s$$

Berikut langkah penyelesaian:

Vektor arah: $a = (2, 1, -1)$ untuk g_1 dan $b = (-1, 2, 1)$ untuk g_2.	B	S
Dua garis sejajar jika $a = k \cdot b$ untuk suatu $k \in \mathbb{R}$. Dari komponen pertama: $2 = -k \Rightarrow k = -2$; dari komponen kedua: $1 = 2k \Rightarrow k = \frac{1}{2}$. Karena nilai k tidak konsisten, kedua garis tidak sejajar.	B	S
Perpotongan dengan menyamakan koordinat: $2t + s = 1$ (i), $t - 2s = 4$ (ii), $t + s = 2$ (iii). Dari (i) − (iii) diperoleh $t = -1$, kemudian $s = 3$.	B	S
Substitusi ke (ii): $-1 - 2(3) = -7 \neq 4$. Sistem tidak konsisten, sehingga kedua garis tidak berpotongan	B	S
Karena kedua garis tidak sejajar dan tidak berpotongan, maka kedua garis BERIMPIT.	B	S

SOAL 3 Kedudukan Bola dan Bidang Rata

Bola dengan persamaan $x^2 + y^2 + z^2 - 4x + 6y - 2z - 11 = 0$ memiliki pusat M(2, −3, 1) dan jari-jari $r = 5$	B	S
Syarat perlu dan cukup agar bidang rata α menyinggung bola B(M, r) adalah jarak dari pusat M ke bidang α sama dengan r.	B	S
Bidang rata α: $x + 2y - 2z = 12$ memotong bola $x^2 + y^2 + z^2 = 16$ pada sebuah lingkaran.	B	S
Jika sebuah bidang rata memotong bola dan melalui pusat bola, maka irisan antara bidang tersebut dengan bola disebut lingkaran besar (great circle) dan jari-jari lingkaran irisannya sama dengan jari-jari bola.	B	S

Figure 1. Solid Analytic Geometry UAS Problems

Each statement/step was mapped onto one or more mathematical reasoning indicators (IPM), adapted from the Reasoning and Proof process standard in the Principles and Standards for School Mathematics (NCTM, 2000), which emphasizes that competent mathematical

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reasoners recognize patterns and regularities, investigate whether such patterns occur by chance or for an underlying reason, make and investigate conjectures, and develop and evaluate mathematical arguments and proofs. The four indicators, together with their theoretical basis, are presented in Table 1 below.

Table 1. Mathematical Reasoning Indicators (IPM)

Code	Mathematical Reasoning Indicator	Reasoning and Proof
IPM-1	Performing mathematical manipulation	Formalizing mathematical proof/argument is a formal way of expressing reasoning; in Solid Analytic Geometry, this formal argument is constructed through accurate algebra-vector manipulation.
IPM-2	Constructing proof/giving reasons for the validity of the solution	Students develop mathematical arguments and proofs (develop mathematical arguments and proofs).
IPM-3	Drawing conclusions from a statement	Through exploration and justification of results, students arrive at the conclusion that mathematics makes sense (mathematics makes sense).
IPM-4	Examining the validity of an argument	Able to investigate whether a pattern occurs coincidentally or for an underlying reason, and to evaluate mathematical arguments and proofs (evaluate mathematical arguments and proofs).

Data analysis in this study followed the six stages of qualitative data analysis proposed by Creswell (2016), adapted to the characteristics of the UAS score data, as follows.

1. Organizing and preparing the data for analysis. The UAS answer sheets of the 32 students were collected and compiled into a digital tabular format based on each statement/step in Problems 1–3; students' identities were anonymized into codes (M-01 to M-32) to protect personal data confidentiality.
2. Reading through the entire data set. All students' answers and arguments for each item were read thoroughly to obtain a general picture of the variation in responses and the patterns of answers that emerged.

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3. Coding the data. Each statement/step was scored on a 0–4 scale based on the assessment rubric (accuracy of the answer and quality of the argumentation), and simultaneously coded according to the mathematical reasoning indicator it measured (IPM-1, IPM-2, IPM-3, IPM-4) as mapped in Table 1.
4. Applying the coding process to generate categories. Each student's coded scores were averaged per indicator and then classified into three achievement categories: high (3.1–4.0), medium (2.1–3.0), and low (0–2.0).
5. Presenting the description and categories in narrative form. The category distribution and achievement patterns for each indicator were presented in tables and described narratively to identify the achievement tendencies of each indicator as well as the overall category distribution of students' mathematical reasoning ability (see the Results section).
6. Interpreting the meaning of the data. Achievement patterns across indicators were interpreted by relating them to theoretical studies and relevant research findings to gain an in-depth understanding of students' mathematical reasoning characteristics and their implications for instruction (see the Discussion section).

RESULTS AND DISCUSSION

Based on the scoring results of the 32 students, the overall mean mathematical reasoning ability was 2.92, which falls into the medium category, close to the boundary of the high category. The category distribution of students' overall mathematical reasoning ability is presented in Table 2 below:

Table 2. Distribution of Students' Mathematical Reasoning Ability Categories

Category	Number of Students	Percentage
High	15	46.9%
Medium	14	43.8%
Low	3	9.4%
Total	32	100%

The majority of students (46.9%) were in the high category; however, the proportion in the medium category was also considerable (43.8%), indicating that students' mathematical reasoning achievement remains varied and unevenly distributed.

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The average achievement per mathematical reasoning indicator is presented in Table 3 below:

Table 3. Average Student Achievement per Mathematical Reasoning Indicator

Indicator	Mean	Category	Rank
IPM-2 (Constructing proof/giving reasons)	3.13	High	1
IPM-4 (Examining the validity of arguments)	3.00	Medium*	2
IPM-3 (Drawing conclusions)	2.95	Medium	3
IPM-1 (Mathematical manipulation)	2.61	Medium	4

Based on Table 3, the mean achievement of students across the four mathematical reasoning indicators shows a fairly clear variation. The IPM-2 indicator (constructing proof/giving reasons) obtained the highest mean of 3.13 in the high category, ranking first. The IPM-4 indicator (examining the validity of arguments) ranked second with a mean of 3.00, exactly at the boundary between the medium and high categories. Meanwhile, IPM-3 (drawing conclusions) and IPM-1 (mathematical manipulation) ranked third and fourth, with means of 2.95 and 2.61, respectively, both remaining in the medium category. Overall, these results indicate that students are relatively more capable of constructing proofs and giving logical reasons than of performing mathematical manipulation procedurally, which in fact is the indicator with the lowest achievement. The category distribution for each indicator is presented next.

Table 4. Distribution of Student Categories per Mathematical Reasoning Indicator

Indicator	Low	Medium	High
IPM-1	6 (18.8%)	15 (46.9%)	11 (34.4%)
IPM-2	4 (12.5%)	12 (37.5%)	16 (50.0%)
IPM-3	5 (15.6%)	11 (34.4%)	16 (50.0%)
IPM-4	2 (6.3%)	15 (46.9%)	15 (46.9%)

Table 4 presents the category distribution of students on each mathematical reasoning indicator. On the IPM-1 indicator (mathematical manipulation), most students were in the medium category (46.9%), followed by the high category (34.4%), while 18.8% remained in the

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low category—the largest proportion among all indicators. The IPM-2 indicator (constructing proof/giving reasons) showed a better distribution, with 50.0% of students in the high category and only 12.5% in the low category, consistent with this indicator's highest mean achievement. For IPM-3 (drawing conclusions), although 50.0% of students were in the high category, 15.6% remained in the low category. Meanwhile, IPM-4 (examining the validity of arguments) showed a relatively even distribution between the medium (46.9%) and high (46.9%) categories, with the smallest proportion in the low category (6.3%) among all indicators. Overall, this distribution reinforces the findings in Table 3, in which IPM-1 has the highest proportion of students in the low category, indicating that mathematical manipulation ability remains the aspect most in need of attention in solid analytic geometry instruction.

Students demonstrated relatively good ability in constructing arguments or giving reasons for the validity of a solution (mean = 3.13, high category, with 50% of students in the high category). This was particularly evident in Problem 2, in which students were asked to identify an error in the step of determining the position of two lines and to provide a correction along with the reasoning. This finding is consistent with the view of Herbert et al. (2022) that the ability to construct justifications is a reasoning component that can develop reasonably well when students are habitually confronted with tasks that explicitly require argumentation, as in the True/False items with reasoning used in this instrument. This is also in line with the finding of Mata-Pereira & da Ponte (2017) that instructional actions that consistently require students to justify their answers can foster the development of such justification ability. This finding indicates that students are reasonably able to communicate reasoning verbally/in writing, although—as will be discussed next—their computational/manipulation ability remains weak.

This indicator recorded the lowest mean (2.61) with the largest proportion of students in the low category among the four indicators (18.8%). Mathematical manipulation in this context is closely related to algebra-vector operations, such as determining the parallelism of two direction vectors (Problem 2) and calculating the distance from the center of a sphere to a plane (Problem 3). This weakness is consistent with the findings of Asmaun et al. (2025), who identified transformation errors and process-skill errors as the dominant types of errors in solving Analytic Geometry problems, as well as with the findings of Noto et al. (2016) regarding students' weak mathematical representation ability in analytic geometry material. This is also consistent with the view of Clements & Battista (1992) that success in geometric reasoning depends heavily on the ability to accurately manipulate spatial representations into symbolic/algebraic form—a stage that demands procedural precision rather than mere conceptual understanding. This weakness indicates that students still experience difficulty at the procedural/computational

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stage, even though they show relatively good conceptual understanding (constructing arguments, IPM-2).

The achievement pattern of IPM-2 (high) versus IPM-1 (lowest) indicates a gap between students' conceptual understanding and their procedural/computational ability. Students tend to find it easier to explain 'why' a statement is true or false conceptually, yet experience difficulty when required to perform algebra-vector manipulation accurately to support that argument. A similar gap between conceptual and procedural knowledge in mathematical reasoning has also been reported by Rodrigues et al. (2021) and by Downton & Livy (2022) in the context of students' geometric reasoning about prisms, underscoring that both aspects need to be developed in a balanced manner and cannot be assumed to develop automatically from one aspect alone.

There were 3 students (9.4%) in the overall low category who consistently showed low scores across all indicators (rather than on a specific indicator only), suggesting a possible comprehensive foundational difficulty with Solid Analytic Geometry material, rather than merely a weakness in a specific reasoning aspect. This finding reinforces the relevance of metacognitive-based intervention approaches, such as the MURDER technique applied by Asnawati & Dewi (2016) in an Analytic Geometry course, as an alternative mentoring strategy for students with low reasoning achievement.

These findings suggest the need to strengthen practice in mathematical manipulation (vector operations, systems of linear equations for determining the position of lines/planes) through structured and gradual exercises, while maintaining the instructional approaches that have proven reasonably effective in training students' conceptual argumentation ability, such as the True/False items with justification used in this instrument. A formative assessment rubric based on reasoning indicators, as developed by Herbert et al. (2022), could serve as a tool for lecturers to continuously monitor students' achievement development on each indicator throughout the course.

CONCLUSION

Students' mathematical reasoning ability in solving Solid Analytic Geometry problems was, overall, in the medium category (mean = 2.92), with 46.9% of students in the high category, 43.8% in the medium category, and 9.4% in the low category. In terms of individual indicators, the ability to construct proof/give reasons (IPM-2) was the indicator with the highest achievement (3.13, high), while the ability to perform mathematical manipulation (IPM-1) was the indicator with the lowest achievement (2.61, medium). This pattern indicates a gap between

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students' conceptual understanding and procedural ability, particularly in the algebra-vector operations underlying the solution of solid analytic geometry problems. In addition, a small group of students (9.4%) was found to experience comprehensive difficulty across all indicators, requiring specific mentoring.

Based on these findings, it is recommended that instruction in Solid Analytic Geometry strengthen algebra-vector manipulation practice in a structured and gradual manner, without neglecting instructional strategies that have proven effective in training students' conceptual argumentation ability. Future research is recommended to complement the UAS score data with in-depth interviews to trace students' thinking processes in more detail, and to test the effectiveness of specific instructional strategies (e.g., a metacognitive approach) in improving achievement on reasoning indicators that remain low.

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