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Bridging Concrete And Digital Manipulatives: Akar Quest For Teaching Square Roots In Low-Resource Classrooms

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Abstract

Many Indonesian lower-secondary students memorise square-root rules without understanding them and copy answers from artificial-intelligence tools, while their classrooms often lack projectors. Research on manipulatives shows benefits for conceptual learning, but most evidence on blending physical and virtual manipulatives comes from well-resourced settings and from topics other than square roots. This study examined the learning process and conceptual understanding of students taught with Akar Quest, a low-cost hybrid manipulative that pairs a wooden grid board and unit tiles with a smartphone web page opened through a QR code. A posttest-only quasi-experiment compared two intact Grade VIII classes at a public junior secondary school in Gowa Regency, Indonesia, supported by classroom observation. The Akar Quest class outperformed the comparison class (M = 81.14 vs. 45.45; Mann–Whitney U = 389.5, p < .001; Hedges' g = 1.42), and more of its students reached mastery (66.7% vs. 18.2%). The largest observed differences were in explaining the concept in one's own words and linking tile arrangements to notation. The study contributes a projector-free design in which students' own phones carry the digital layer of a manipulative. The results are preliminary but suggest a practical route to conceptual learning where classroom technology is scarce.

Keywords: Hybrid Manipulatives, Square Roots, Conceptual Understanding, Mobile Learning, Low-Resource Classrooms

INTRODUCTION

Digital technology is often promoted as a route to better learning in developing

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countries, yet many classrooms lack the infrastructure that technology-rich lessons assume. The 2023 Global Education Monitoring Report argues that technology in education should be adopted only where it is appropriate, equitable, scalable, and sustainable, and that it should complement rather than replace face-to-face teaching (UNESCO, 2023). In Indonesia, public lower-secondary schools outside major cities frequently have no classroom projectors, while most students own or can access a smartphone. Similar conditions elsewhere have led teachers in under-resourced secondary schools to use mobile devices in place of the computers their schools lack (Parajuli et al., 2024). The same phones, however, also offer shortcuts: over-reliance on AI dialogue systems can weaken students' critical and analytical thinking because learners favour fast cognitive shortcuts (Zhai et al., 2024).

A preliminary study by the authors at a public junior secondary school in Makassar showed this pattern directly. Teachers reported that students were weak in multiplication facts, memorised formulas without understanding, and copied answers from AI applications, then struggled on independent tests; the classrooms had no projectors. Square roots illustrate the problem well. Students can write √25 = 5 without recognising that the statement answers a simple geometric question: what is the side length of a square whose area is 25 square units? Earlier work in this journal showed that grounding mathematics in meaningful contexts can improve students' problem solving (Nasrullah, 2026), yet square roots are usually taught as symbol manipulation. Errors such as √9 + √16 = √25 are typical of procedures applied without conceptual grounding, and they persist among Indonesian lower-secondary students (Phibeta & Wijayanti, 2022; Setyaningtyas et al., 2018).

Research on manipulatives and on blending physical and virtual materials offers a promising response, but, as the literature review below shows, most of this evidence comes from well-resourced settings and from topics such as whole numbers, fractions, and science. Little is known about low-cost hybrid manipulatives for square roots in classrooms where the only available screen is the student's own phone. To address this gap, a team of pre-service teachers developed Akar Quest in a project-based manipulative learning media course, a setting in which opportunity to learn has been linked to pre-service teachers' growth mindset (Nasrullah et al., 2025). Akar Quest combines a wooden grid board and unit tiles with a mobile-first web page reached by scanning a QR code. Students first build squares with tiles and then see the same relationship visualised and written as $\sqrt{L} = s$ on their own phones. The novelty of the design is twofold: it moves the digital layer of a manipulative onto students' phones so that no projector is needed, and it redirects phone use towards tasks that require students to construct an answer physically before seeing it symbolically. This article reports a classroom trial of Akar Quest guided

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by three questions: (1) How did learning unfold with and without Akar Quest? (2) How did conceptual understanding compare between the two classes? (3) What strengths and constraints emerged that should inform revision and wider use in low-resource classrooms?

LITERATURE REVIEW

Conceptual Understanding of Square Roots

Conceptual and procedural knowledge develop together, and procedures without conceptual roots tend to be fragile (Hiebert & Lefevre, 1986; Rittle-Johnson et al., 2015); conceptual understanding is therefore treated as a core strand of mathematical proficiency (Kilpatrick et al., 2001). In Indonesian research, conceptual understanding is commonly assessed through indicators such as restating a concept, classifying objects, giving examples and non-examples, representing a concept in multiple forms, and applying procedures and concepts, and lower-secondary students often meet only some of them (Fajar et al., 2019). Studies of Grade 8 students report persistent difficulties in identifying perfect squares, relating perfect squares to their roots, and operating on square roots (Can, 2024; İşleyen & Mercan, 2013). Diagnostic studies in Indonesia similarly document misconceptions about the concept and properties of radicals among junior secondary students (Setyaningtyas et al., 2018) and about the properties of radicals and rationalisation (Phibeta & Wijayanti, 2022). These studies describe the difficulties well, but few test instructional designs that give square roots a concrete referent.

Manipulatives and the Concrete–Representational–Abstract Sequence

Manipulatives offer a way to build meaning before symbols. The idea draws on Bruner's (1966) enactive–iconic–symbolic modes of representation and is operationalised in the concrete–representational–abstract (CRA) sequence, classified as an evidence-based practice for students who struggle in mathematics (Agrawal & Morin, 2016; Bouck et al., 2018; Witzel et al., 2003). Research on concreteness fading shows that moving progressively from concrete to idealised and then symbolic representations supports transfer better than concrete or abstract materials alone (Fyfe et al., 2014). Meta-analyses show small-to-moderate benefits of concrete manipulatives over symbol-only instruction, but the effect depends strongly on how they are used (Carbonneau et al., 2013; Sowell, 1989). A systematic review found interventions with manipulatives effective for a range of objectives, including conceptual understanding, while noting methodological weaknesses (Lafay et al., 2019), and a scoping review confirmed that physical manipulatives promote children's active involvement in hands-on learning (Byrne et al., 2023). In Indonesia, manipulative media improved Grade VIII students' conceptual

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understanding of circles (Murniasih, 2018), a locally made manipulative supported junior secondary students' addition and subtraction of integers (Amirullah & Nasrullah, 2018), and a braille-based factorisation board was effective for teaching least common multiples and greatest common factors to visually impaired students (Sahid et al., 2026). Objects do not carry meaning by themselves, however: students must come to see them as representations of mathematical ideas (Uttal et al., 1997), and enjoyable activity does not guarantee understanding unless the teacher connects objects to concepts (Laski et al., 2015; Moyer, 2001).

Virtual, Hybrid, and Mobile Learning Materials

Virtual manipulatives add affordances that physical objects lack, such as simultaneous linking of images and notation and immediate feedback, and a meta-analysis reported a moderate effect on achievement (Moyer-Packenham & Westenskow, 2013). Blending physical and virtual manipulatives according to the strengths of each can produce better conceptual understanding than either alone (Olympiou & Zacharia, 2012), because physical manipulation and digital representation offer different representational pathways (Manches & O'Malley, 2012). A recent meta-analysis of 35 studies found no overall advantage of physical over virtual investigation for conceptual knowledge, with virtual investigation preferable mainly when touch offers no relevant information about the concept (Muilwijk & Lazonder, 2023). The choice should therefore follow the concept: where handling objects carries meaning, as when tiles are arranged into a square, the physical stage is worth keeping. On the digital side, an HTML-based worksheet validated by experts was found to support active mathematics learning (Nasrullah et al., 2021), and QR-code-assisted mathematics worksheets for Grade VIII have been reported as valid and practical (Himmah & Musdi, 2022), and teachers in under-resourced schools turn to mobile devices when computers are absent (Parajuli et al., 2024). Earlier work by the first author also used culturally familiar, concrete activities, including traditional games for early counting (Nasrullah & Zulkardi, 2011) and a hypothetical learning trajectory for integer operations (Moanoang et al., 2021). What remains missing is evidence on a low-cost design that keeps the physical stage and delivers the representational stage through students' own phones for a topic such as square roots; this study takes that position.

RESEARCH METHOD

Research Approach and Design

The study used a mixed approach: a field trial with a posttest-only quasi-experimental comparison of two intact classes (quantitative), complemented by qualitative classroom

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observation. This design was chosen because the school did not permit random assignment and the trial aimed to assess the media under real classroom conditions. The study ran from 18 August to 10 November 2026 in two phases. The development phase began on 18 August with idea generation and continued with a school visit to confirm the idea with the mathematics teacher, brainstorming, a design presentation with expert feedback, construction of the media, and a pilot run. The implementation phase took place on 10 November 2026, when each class received one 120-minute lesson and the posttest.

Setting, Respondents, and Data Sources

The study was conducted at a public junior secondary school in Gowa Regency, South Sulawesi, Indonesia, a semi-urban area adjoining the provincial capital, where classrooms had no projectors. Respondents were Grade VIII students in two classes assigned by the school (purposive selection of intact classes). The Akar Quest class (VIII.4) had 40 enrolled students (18 boys, 22 girls) and the comparison class (VIII.6) had 38 (18 boys, 20 girls). Quantitative data were students' test scores; only students who sat the test were analysed (21 and 22, respectively). Qualitative data were observation ratings and field notes recorded by one observer per class. The same instructor, a pre-service mathematics teacher from the development team, taught both classes with the same objectives, content, and test; only the use of the media differed.

The Akar Quest Media and Instructional Framework

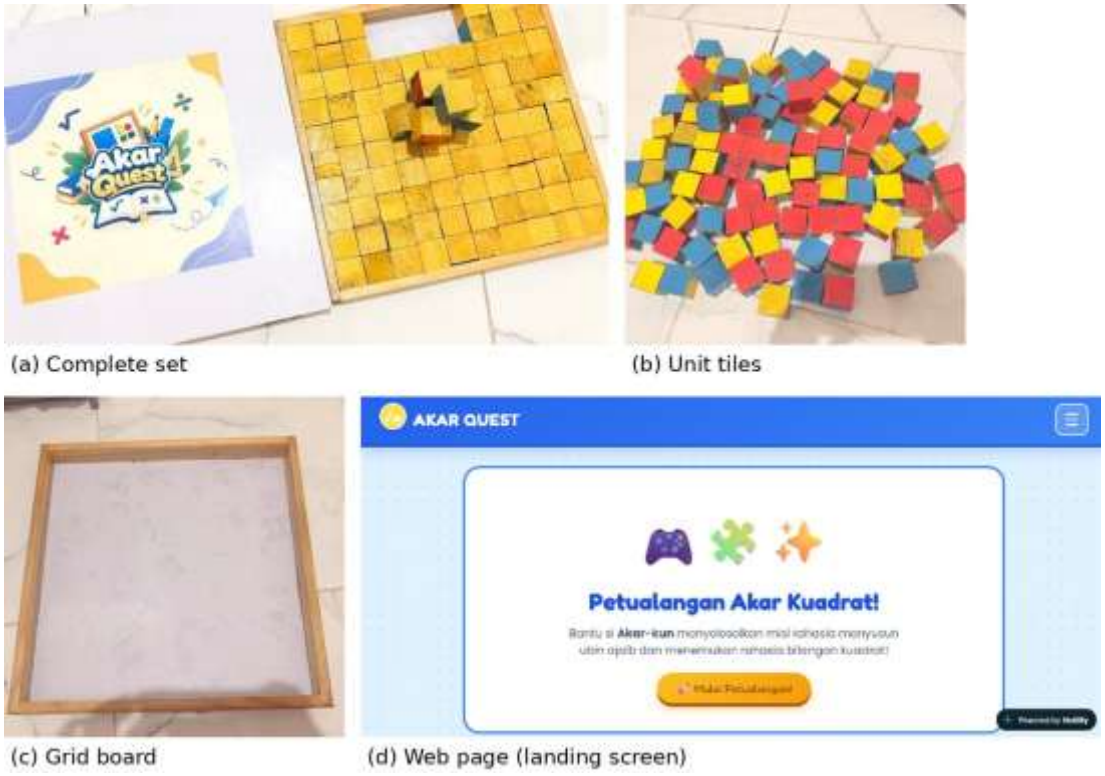
Akar Quest has physical and digital components (Figure 1). The physical set comprises a 10 × 10 wooden grid board and 100 coloured wooden unit tiles (4 cm × 4 cm) in a storage box. During expert review, a mathematics education lecturer recommended replacing the original plywood board with sturdier material and making students build the concept with concrete objects before moving to exercises; both recommendations were adopted. The digital component is a mobile-first web page opened by scanning a QR code on the box. It contains a learning zone that visualises the relationship between area and side length with the notation $L = s \times s$ and $s = \sqrt{L}$, a puzzle mission, and a ten-item constructed-response quiz with automatic feedback and worked solutions. Because it runs on students' phones, the media requires no projector, computer laboratory, or app installation.

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(a) Complete set (b) Unit tiles

(c) Grid board (d) Web page (landing screen)

Figure 1. Components of Akar Quest

The lesson design followed the concrete–visual–symbolic framework derived from Bruner (1966) and the CRA sequence in five steps. First, the teacher posed a prompt without any formula: "If you have 25 unit tiles, can you arrange them on the grid to form a complete square?" Second, students worked in groups to build a 5 × 5 square and identified the number of tiles as the area and the number of tiles along one side as the side length. Third, students scanned the QR code and viewed the visualisation and the notation √25 = 5. Fourth, students completed the puzzle mission and quiz with automatic feedback. Fifth, the teacher led a reflection in which students concluded that the square root of a perfect-square area gives the side length of the square. The comparison class received direct explanation and worked examples, followed by practice and consolidation.

Instruments and Data Collection

The conceptual understanding test had an exploration section (completing a table of tile arrangements for areas of 16, 25, and 36; 20 points) and ten constructed-response items (8 points each; 80 points), for a maximum of 100. Items targeted six indicators of conceptual understanding used in Indonesian mathematics education research (Fajar et al., 2019; Kilpatrick

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et al., 2001): restating a concept, classifying objects by properties, giving examples and non-examples, representing the concept in multiple forms, applying procedures or operations, and applying the concept in problem solving. Sample items asked students to find the side length of a square board with an area of 36 square units, to decide whether 49 or 50 tiles can form a square, and to compute √9 + √16. The observation sheet contained ten aspects of student activity rated from 1 (poor) to 4 (very good), with space for field notes. The test was administered at the end of each lesson, and observation was conducted throughout. The school's mastery criterion (*kriteria ketercapaian tujuan pembelajaran*, KKTP) was a score of 75.

Data Analysis

Test scores were summarised with means, medians, standard deviations, ranges, and the proportion of students reaching mastery. Normality was checked with the Shapiro–Wilk test and variance homogeneity with Levene's test. Because scores in the Akar Quest class were not normally distributed ($W = 0.84$, $p = .003$), the Mann–Whitney U test was the primary test, with Welch's t test as a robustness check. Effect sizes are reported as Hedges' g with a 95% confidence interval and as the rank-biserial correlation, interpreted following Cohen (1988). Mastery rates were compared with Fisher's exact test. Calculations were run in Python with the assistance of Claude (Anthropic) from the individual score data and verified by the authors. Observation ratings were converted to percentages of the maximum score, and field notes were analysed thematically to describe the learning process.

Data Credibility

Credibility was supported by triangulating test scores with observation ratings and field notes, by using the same instructor, objectives, content, and test in both classes, and by checking parametric results against a non-parametric test. Threats that could not be removed, such as intact groups, a single observer per class, and the absence of reliability and validity evidence for the test, are reported in the Discussion. The trial was conducted with the permission of the participating school, and student data are reported anonymously.

RESULTS

The Learning Process

In the Akar Quest class, students showed interest as soon as the media was introduced. Groups arranged tiles on the grid, observed that 25 tiles formed a square of five rows and five columns, and then matched this finding to the visualisation and notation on the web page. The

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observer recorded a cheerful atmosphere, active engagement with both physical and digital components, and no technical problems; the field notes record that almost all students competed to answer the challenge questions. In the comparison class, instruction centred on the teacher's explanation and worked examples. Students attended well, but the observer noted that not all were willing to come forward and solve problems on the board. Table 1 summarises the observation ratings.

Table 1. Observed Classroom Activity Ratings (Scale 1–4)

Aspect observed	**Akar Quest**	**Comparison**
1. Attending to the teacher's explanation	4	3
2. Actively asking or answering questions	4	3
3. Collaborating well in groups	4	3
4. Direct involvement (manipulating media or doing exercises)	4	4
5. Explaining square roots in their own words	3	2
6. Showing enthusiasm and curiosity	4	3
7. Completing evaluation tasks independently	4	4
8. Staying focused on learning activities	4	3
9. Linking concrete or visual experience with notation	3	2
10. Lesson ran smoothly and on time	4	4
Total (percentage of maximum)	**38 (95.0%)**	**31 (77.5%)**

Note. Ratings from one observer per class.

The Akar Quest class was rated higher on seven of ten aspects. The two aspects closest to conceptual understanding, explaining the concept in one's own words and linking concrete or visual experience to notation, were rated 3 in the Akar Quest class and 2 in the comparison class. The equal ratings for direct involvement, independent work, and lesson flow indicate that both lessons were orderly and completed on time.

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Conceptual Understanding

Table 2 presents descriptive statistics for the test.

Table 2. Descriptive Statistics for the Conceptual Understanding Test

Statistic	Akar Quest	Comparison
Students tested	21	22
Mean	81.14	45.45
Median	88.0	42.0
Standard deviation	20.75	27.84
Range	36–100	2–100
Reached mastery (≥ 75)	14 (66.7%)	4 (18.2%)

Note. Computed from individual scores of students present for the test.

The Akar Quest class scored 35.69 points higher on average. Half of its students scored 88 or above, whereas the comparison class median was 42. Scores in the comparison class were also more dispersed: four students scored between 2 and 18, while the lowest score in the Akar Quest class was 36. The difference was statistically significant, Mann–Whitney $U = 389.5$, $p < .001$, rank-biserial $r = .69$. Welch's test led to the same conclusion, $t(38.8) = 4.78$, $p < .001$. The effect was large, Hedges' $g = 1.42$, 95% CI [0.75, 2.09]. Mastery rates also differed significantly (Fisher's exact test, $p = .002$), with odds of reaching mastery about nine times higher in the Akar Quest class (odds ratio = 9.0). Scores were not analysed by indicator because item-level scores were not available for all students; the comparison therefore rests on total scores.

Strengths and Constraints

The strengths identified were students' direct involvement in building squares, the combination of concrete and digital experience, and the ability to run the lesson without a projector because digital content opened on students' phones. The constraints were practical. Teachers must prepare boards and tiles and check that the web page loads; phone use depends on device availability and connectivity; and turn-taking must be organised so that every student handles the tiles. Direct instruction was simpler to prepare, but visualisation of the concept depended entirely on the teacher's explanation.

DISCUSSION

Students who learned square roots with Akar Quest scored substantially higher on a conceptual understanding test than students taught through direct explanation, with a large

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effect. The direction of this result is consistent with meta-analyses of concrete and virtual manipulatives (Carbonneau et al., 2013; Moyer-Packenham & Westenskow, 2013) and with Indonesian evidence that manipulative media support conceptual understanding (Murniasih, 2018), although the magnitude exceeds the averages those reviews report. Effects this large are common in small trials that use researcher-developed tests closely aligned with the intervention, so the estimate should be treated with caution.

The most plausible explanation lies in the sequence of representations. Arranging 25 tiles into a 5 × 5 square gives students an enactive referent for the symbol √25 = 5, as Bruner's (1966) framework, the CRA sequence, and concreteness fading propose (Agrawal & Morin, 2016; Fyfe et al., 2014; Witzel et al., 2003). Because the act of arranging tiles itself carries the meaning of area, this is a case in which the physical stage is worth keeping rather than replacing with a simulation (Muilwijk & Lazonder, 2023). The web page then serves as the representational stage, linking the tile arrangement to notation simultaneously, a key affordance of virtual manipulatives (Moyer-Packenham & Westenskow, 2013). The observation data support this reading: the aspects that most distinguished the classes were explaining the concept in one's own words and connecting concrete experience to notation, not general activity. The media appears to have helped students build the bridge between object and symbol that Uttal et al. (1997) identify as the condition for manipulatives to be meaningful.

The design also targets a well-documented misconception. Adding the numbers under the radical sign, as in √9 + √16 = √25, is common among Grade 8 students (İşleyen & Mercan, 2013), and related misconceptions about radicals are documented among Indonesian students (Phibeta & Wijayanti, 2022; Setyaningtyas et al., 2018). Students who interpret √9 and √16 as the side lengths of two different squares, 3 and 4, have a conceptual reason to add 3 + 4 rather than 9 + 16, consistent with the view that conceptual and procedural knowledge reinforce each other (Rittle-Johnson et al., 2015). Whether Akar Quest reduces this specific error should be tested with item-level analysis in future trials.

The findings add two contributions to work on technology in low-resource classrooms. First, the absence of projectors need not exclude students from visual, interactive representations; moving the digital layer to students' own phones made it available at almost no cost to the school, extending reports of mobile devices replacing absent computers (Parajuli et al., 2024) and of QR-code-assisted worksheets (Himmah & Musdi, 2022), and aligning with UNESCO's (2023) call for technology that is appropriate, equitable, and complementary to the teacher. Second, the response to AI dependence was not to ban technology but to redirect the phone towards tasks that require students to construct an answer physically first. Blending

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physical and digital materials according to their strengths (Olympiou & Zacharia, 2012) may be especially valuable where neither alone is sufficient. Enthusiasm, however, is not understanding (Moyer, 2001); the teacher's role in the reflection step remains essential (Laski et al., 2015). Bring-your-own-device designs also assume working phones and data, so scaling the approach to poorer or more remote schools will require offline versions of the web page, shared devices, or printed visual cards.

Several limitations constrain causal claims. The posttest-only design with intact classes cannot rule out pre-existing differences between classes. The intervention lasted one lesson and was taught by a member of the development team, which risks expectancy and novelty effects. Nearly half of the enrolled students were absent on the test day (19 of 40 in the Akar Quest class and 16 of 38 in the comparison class), so the tested samples may not represent their classes. The test was developed by the team, included a tile-based exploration section more familiar to the Akar Quest class, and has not been examined for reliability and validity. The worksheet on which the test was printed invited group discussion, and its items resembled those of the web-based quiz, so individual understanding cannot be fully separated from collaboration and item familiarity. Finally, each class was observed by a single observer without inter-rater agreement.

CONCLUSION

This trial shows that a low-cost hybrid manipulative, combining a grid board and unit tiles with a smartphone web page in a concrete–visual–symbolic sequence, can be implemented in a classroom without projectors and is associated with high engagement and stronger conceptual understanding of square roots than direct instruction. Students who used Akar Quest were more likely to reach mastery and were better able to explain the concept and connect concrete experience with notation. Given the single lesson, intact groups, high absence, and an unvalidated test, these findings are promising but preliminary. Further research should use a pretest–posttest design with equivalent groups, several lessons, teachers who are not the developers, validated instruments, a delayed posttest, and item-level error analysis. Future versions of the media should structure group roles so that every student manipulates the tiles, enrich feedback, offer an offline mode, and extend the content to non-perfect squares and simplifying radicals.

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