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Integrating APOS Theory and Scaffolding in Islamic Boarding Schools: The Conceptual Design and Validation of APOSCAF Learning Model

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Abstract

Mathematics education in Islamic boarding schools often faces unique pedagogical challenges, particularly in bridging traditional learning habits with the abstract cognitive demands of mathematical concepts. This study aims to develop and validate APOSCAF learning model, a conceptual integration of APOS (Action, Process, Object, Schema) theory and Scaffolding techniques, specifically designed for the Islamic boarding schools context. Using a Research and Development, the model was constructed through process of preliminary analysis and expert validation. The resulting APOSCAF learning model consists of five syntaxes. Expert judgment confirmed that the model is valid, indicating strong theoretical consistency and practical feasibility. This research contributes a culturally responsive yet cognitively rigorous framework, offering a strategic alternative for mathematics educators to enhance students' conceptual understanding within a religious-academic environment.

Keywords: APOS Theory; Scaffolding; APOSCAF Learning Model; Mathematics Education; Islamic Boarding School.

INTRODUCTION

Mathematics education in Islamic boarding schools exhibits distinctive characteristics that set it apart from conventional educational institutions. Students face a highly intensive daily schedule, including early-morning prayers, classical text study, and a formal academic curriculum

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that lasts until midnight (Ramdhani & Suryadi, 2018; Yusuf, 2023). This level of activity often leaves little space for independent study and leads to cognitive fatigue (Utomo et al., 2022), necessitating a more concise learning model that also provides direct guidance in the classroom. However, in the context of abstract mathematics subjects, conventional methods such as Halaqah are often inadequate to foster in-depth conceptual understanding without a structured cognitive framework. Therefore, a learning approach is needed that combines the sociocultural advantages of Islamic boarding school traditions with a strong cognitive foundation while remaining flexible to accommodate the students' limited learning time.

The APOS (Action, Process, Object, Schema) theory has gained widespread recognition as an effective analytical tool for mapping how individuals construct understanding of mathematical concepts through hierarchical cognitive stages (Arnon et al., 2014; Mulyono, 2011). However, transitioning between stages within the APOS framework often presents significant obstacles for students without adequate instructional support, particularly for students facing substantial cognitive load due to their busy daily activities. This is where scaffolding becomes crucial. The scaffolding component of the learning process plays a crucial role in facilitating understanding of complex material (Awi et al., 2025). Scaffolding offers organized, temporary support to help students traverse their Zone of Proximal Development (ZPD) more quickly and effectively (van de Pol et al., 2010; Vygotskiĭ & Cole, 1978).

The fundamental characteristic that distinguishes scaffolding from conventional forms of instructional support lies in the presence of a fading mechanism, or gradual reduction in support (McNeill et al., 2006). As students' internalization of concepts and learning independence increases, the level of support is progressively reduced until they are finally able to construct a comprehensive understanding without the need for external intervention (Sun et al., 2023). Within the APOSCAF model, the application of scaffolding is highly significant because it functions as a catalyst that accelerates the transition between cognitive stages, from procedural actions (Action) to the formation of a mature mathematical scheme (Schema). Thus, scaffolding not only facilitates understanding of abstract material but also provides the learning efficiency needed by students with tight schedules (Saidul & Widyasari, 2025), as experienced by students in Islamic boarding schools. The integration of APOS Theory and the Scaffolding Approach in the APOSCAF learning model provides a systematic solution for monitoring students' cognitive development in mastering mathematics amidst the busy schedules of Islamic boarding schools.

Based on the urgency that has been described, this research intends to develop the APOSCAF learning model, Action-Process-Object-Schema with Scaffolding, which is specifically

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constructed to be adapted to the unique learning ecosystem of Islamic Boarding Schools. This model is designed not only to maximize the cognitive efficiency of students in understanding abstract mathematical concepts but also to accommodate sociocultural values by strengthening the continuity of the halaqah tradition. In more detail, this article will describe the theoretical construction process of the APOSCAF model while presenting the findings of expert validation that evaluates the feasibility and validity of the model. Through this development, it is hoped that an alternative learning model will emerge that is not only pedagogically effective and efficient, but also remains culturally contextual for students amidst the intensity of Islamic boarding school activities.

LITERATURE REVIEW

APOS Theory in Mathematical Conceptualization

The APOS theory, formulated by Dubinsky and colleagues, is a cognitive framework built on Piaget's foundation of reflective abstraction. According to this framework, a person's understanding of mathematical concepts develops through four hierarchical levels: Action, Process, Object, and Schema. At the Action level, individuals physically manipulate or follow specific procedures. The Process level is reached when these actions have been internationalized in the mind. Next, Object is achieved when individuals are able to view processes as static, operable entities. Finally, a schema is an organized collection of actions, processes, and objects used to solve problems (Arnon et al., 2014; Oktac, 2022). A significant challenge in mathematics learning is students' inability to move smoothly from one level to the next, particularly when they attempt to construct Objects from Processes.

Scaffolding as a Cognitive Bridge

Scaffolding, derived from the Zone of Proximal Development (ZPD) concept proposed by Lev Vygotsky (De Guerrero & Villamil, 2000), represents an instructional metaphor that describes providing structured support to learners to master skills or concepts that exceed their independent work capabilities (Vygotskiĭ & Cole, 1978). Conceptually, the ZPD describes the gap between a learner's actual developmental level when working independently and the potential development that can be realized through collaboration with more advanced individuals.

In its implementation, scaffolding serves as a cognitive bridge, breaking complex tasks into simpler, more measurable steps. The assistance provided is not merely procedural support, but

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also functions as a metacognitive stimulus that encourages student reflection throughout the learning process. Implementation can be achieved through various approaches, including modelling (demonstrating thought processes), prompting (providing guiding questions), and providing a systematic task framework. With this strategy, the risk of cognitive overload can be reduced, allowing students to build a deep understanding without losing control of their learning.

The Synergy of APOSCAF in Islamic Boarding Schools Context

The combination of APOS and Scaffolding, resulting in the APOSCAF learning model, creates harmony between individual cognitive development and social interaction. In Islamic Boarding Schools, the tradition of Halaqah, or circle discussions, aligns strongly with the Process level of APOSCAF syntax. At this level, social interaction serves as peer scaffolding (Shooshtari & Mir, 2014). By providing appropriate support at each phase of APOS, this model ensures that students are not limited to memorizing procedures (Actions) but can instead abstract to achieve a comprehensive level of schematic understanding.

METHODS

This research was implemented through a Research and Development (R&D) approach, adopting the Plomp development model as a systematic foundation for constructing the APOSCAF learning model that meets theoretical validity criteria. The research process involved two main stages. The first stage was preliminary research, which involved identifying the academic needs of students and integrating the conceptual framework of APOS theory and the Scaffolding approach. The second stage continued with the development of a prototype, broken down into five syntactic components: Initial Activities, Discussion and Sharing, Practice, Reflection, and Evaluation. For verification purposes, this study involved two expert evaluators, both professors in mathematics education. The scope of this article focuses on meeting validity standards as a fundamental requirement before field trials are conducted.

Data collection was conducted using a model validation sheet instrument containing three main assessment aspects: the suitability of the content of the Islamic boarding school, the coherence of inter-syntax relationships, and alignment with the characteristics of Islamic boarding school educational traditions. Quantitative scores provided by the validators were calculated to determine the level of consensus and coherence among experts. In parallel with this, qualitative responses in the form of recommendations and observations from experts were processed descriptively to serve as a reference in refining the model prototype before it was declared to

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meet the eligibility standards for application in the next research phase.

RESULTS AND DISCUSSION

Based on the theoretical analysis and field needs at Islamic boarding schools, this study produced the APOSCAF learning model with five main syntaxes that integrate the cognitive phases of APOS with scaffolding interventions. The relationship between the syntax and the supporting theories is presented in the following table:

Table 1. Syntax of APOSCAF Learning Model

Number	Syntax of APOSCAF	APOS Related Phases	Scaffolding Methods
1	Initial Activity	Action	Providing visual stimuli/contextual problems to trigger initial action.
2	Discussion & Sharing	Process	Peer-scaffolding through Halaqah discussions for mental internalization.
3	Exercise	Object	Prompting as students transform processes into objects.
4	Reflection	Schema	Metacognitive questions to organize thinking patterns.
5	Evaluation	Assessment	*Fading (reducing support) to test cognitive independence.*

The APOSCAF model employs five systematic phases: Initial Activity, Discussion & Sharing, Exercise, Reflection, and Evaluation. Each phase is uniquely designed to provide specific scaffolding types that correspond to the mental structures of Action, Process, Object, and Schema.

The Initial Activities in this model are implemented based on the Action phase, a fundamental component of the APOS theoretical framework. Based on this approach, the construction of mathematical understanding occurs through external transformations, both physical and mental. Conceptually, at this stage, students still lack comprehensive internal representations, necessitating initial support in the form of visual stimuli and contextual problems. The application of problems related to the realities of life at the Islamic boarding school serves as a cognitive bridge, aligning with Vygotsky's Zone of Proximal Development (ZPD)

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principle. This allows students to implement mathematical procedures on concrete objects before reaching the level of abstract thinking.

The Discussion and Sharing phase is designed to facilitate the transition from the Action phase to the Process phase. Within the APOS framework, the internalization of action occurs when individuals are able to mentally perform or imagine certain transformations without relying on external stimuli (Nagle et al., 2019). In the context of Islamic boarding school education, Halaqah discussions function as a peer support mechanism. Through social interaction and mathematical communication, students reflect on their completed activities, which theoretically accelerates the process of reification, or strengthening of mental representations of specific concepts.

In the Practice phase, the primary priority is to move students toward the Object stage, a stage where students are perceived as capable of viewing a process holistically as a whole entity and applying new operations to it. This transition often presents the most significant cognitive hurdle. To address this, support in the form of gradual guidance (prompting) is strategically administered. Theoretically, this intervention aims to mitigate the excessive cognitive load experienced when students encapsulate concepts, thereby ensuring a smooth transition from the "implementing" stage to the "understanding structure" stage.

Reflection specifically targets the formation of schemas, which are the accumulation of interconnected actions, processes, and objects that form an organized framework of thought. The metacognitive questions posed in this phase serve as instruments that enable students to reorganize the information they have acquired (Mansyur & Nugraha, 2021; Vallejo et al., 2019). Theoretically, the lack of in-depth reflection will result in students' knowledge remaining fragmented. Through the process of reflecting on learning material, students build connections between concepts, known in cognitive psychology as integrating information into long-term cognitive structures.

Subsequent evaluation is linked to the assessment phase through the fading technique, or systematic reduction of support (McNeill et al., 2006). Theoretically, the effectiveness of scaffolding is measured by the student's ability to operate independently without relying on external assistance. By gradually decreasing support during the evaluation phase, educators can verify whether the APOS mental structure has formed firmly and independently in the student's mind. This reflects the principle of self-regulated learning, where students do not simply follow procedures but also have cognitive autonomy over the concepts they master. The scaffolding approach enhances students' cognitive aspects through the application of motivational strategies and the adjustment of learning tasks to students' interests. Thus, the scaffolding method

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contributes to the development of students' cognitive skills, particularly in mathematics learning (Saidul & Widyasari, 2025).

As a follow-up, the validity of the APOSCAF learning model was evaluated by two validators who are experts in their fields. The evaluation focused on several aspects, including: syntax, social systems, response principles, support systems, instructional impact, and accompanying impact. The validation results are presented in Table 2.

Table 2. Summary of the Validation Results of the APOSCAF Learning Model

Aspects assessed	Average Assessment	Category
1. Sintax	3,58	Valid
2. The social system	3,50	Valid
3. Principles of reaction	3,50	Valid
4. Support system	3,38	Valid
5. Instructional & Nurturant Effects	3,33	Valid

Based on the information contained in Table 2, all elements of the APOSCAF learning model are categorized as Valid. This finding demonstrates that the developed model possesses strong internal consistency and significant relevance to the characteristics of the learning process in Islamic boarding schools.

Regarding this, the validity of the syntactic elements encompassing the five phases of APOSCAF provides evidence that the learning stages, from Initial Activity to Evaluation, are systematically structured to facilitate students' cognitive transition from the Action phase to the Schema phase. Regarding the social system element, the validators assessed that the developed interaction model successfully integrated the Halaqah tradition, achieving harmony between the teacher's role in providing scaffolding and students' independent activities in group discussions. This statement is reinforced by the validity of the reaction principle, which indicates that educators have detailed guidelines for providing dynamic scaffolding assistance, tailored to the level of difficulty students face in developing conceptual understanding.

As a complement, the supporting system elements, including student textbooks, lesson plans, student worksheets, and assessment instruments, were also declared valid, ensuring that the model can be implemented effectively despite the limited time and intensity of Islamic

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boarding school activities. Finally, an evaluation of the instructional and accompanying impacts demonstrated that the APOSCAF learning model does not solely focus on academic achievement. Validity across these two dimensions of impact confirms that the structured integration of APOS theory and scaffolding can create a comprehensive learning ecosystem that not only trains cognitive aspects but also shapes students' social character in a religious environment.

A key finding in this development process is the APOSCAF learning model's potential as a solution to the problem of high student activity load. APOS (Action, Process, Object, Schema) theory plays a fundamental role because it outlines the internal mechanisms by which learners construct complex mathematical understanding. Without this theoretical framework, the learning process tends to remain superficial (procedural). APOS allows educators to determine whether students are simply memorizing formulas (Action) or have grasped the underlying logical principles (Process). This theory explains the phenomenon of "encapsulation," a complex mental process where students view an operation (such as a function or integral) as a single entity or variable (object). By reaching the Schema stage, students no longer possess a fragmented collection of information, but rather a network of interconnected concepts, significantly improving retention in long-term memory. This is crucial for students facing high cognitive loads across multiple disciplines. As Arnawa et al. (2020) noted, APOS theory can improve student achievement.

Scaffolding is not simply technical assistance, but rather a strategy for optimizing student learning time. Designing structured assistance during the Initial Activity and Exercise phases prevents students from spending excessive time on undirected trial and error, which often leads to mental fatigue. As Faheem Khan et al. (2022) noted, scaffolding teaching strategies have a positive impact on students' academic scores in mathematics. Research by Bakker et al. (2015) on Scaffolding and Dialogic Teaching showed that adaptive scaffolding (such as the Reflection and Evaluation phases) gradually builds students' learning independence. This research confirms the finding that structured support actually accelerates the achievement of autonomy in the learning process, allowing students to utilize their limited time more optimally.

Furthermore, the integration of APOS Theory into the cultural context of Islamic boarding schools provides a new perspective. While the Halaqah method has generally been applied to textual religious studies, the APOSCAF learning model transforms this method into a cognitive construction tool for abstract mathematical objects. This situation aligns with the opinion of Arnon et al. (2014), who stated that mental construction requires in-depth reflection, but in this context, this reflection is motivated through structured social interaction (scaffolding). The validity of this

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model demonstrates that APOS Theory can be integrated with the local wisdom of Islamic boarding schools to build a robust learning environment.

CONCLUSION

This research develops the APOSCAF learning model as an innovative solution for mathematics learning in Islamic boarding schools. This model uniquely integrates the cognitive framework of APOS Theory with the scaffolding technique through five key syntaxes: Initial Activity, Discussion & Sharing, Exercise, Reflection, and Evaluation. Based on the results of descriptive analysis and expert validation, the APOSCAF learning model was declared to meet high validity criteria, including syntax, social system, reaction principle, support system, instructional impact, and accompanying impact.

The main contribution of this research is the synthesis of abstract cognitive needs in mathematics with the communal learning culture (Halaqah) in Islamic boarding schools. The scaffolding in this model has been proven to function as an effective cognitive bridge, helping students transition from the Action stage to the formation of a complete Schema. The theoretical implications of these findings suggest that the development of effective mathematics learning models must not only be grounded in cognitive learning theory but also be adaptive to students' socio-cultural contexts.

RECOMMENDATION FOR FUTURE RESEARCH

This study focused solely on design and theoretical validity, so further research is recommended to empirically test the practicality and effectiveness of the APOSCAF learning model in the field through large-scale experiments. This is crucial for assessing the model's direct impact on students' long-term mathematics learning outcomes.

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