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Didactic Design Of DDR-Based Fraction Learning For Parental Assistance

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Abstract

Fraction learning in elementary school is often challenging due to its abstract nature and the need for strong conceptual understanding. These difficulties are exacerbated when learning support at home is not guided by systematic didactic principles, despite the important role of parental involvement in sustaining learning. This study aims to explore parents' experiences, perceptions, learning obstacles, and needs in assisting children with fraction learning at home, and to design an initial didactic framework that supports parental involvement using the Didactical Design Research (DDR) approach. This qualitative study focuses on the prospective analysis stage of DDR and involves two elementary school teachers and ten parents of fourth-grade students at a public elementary school in Sukabumi Regency. Data were collected through semi-structured interviews and document analysis, and analyzed thematically to identify learning obstacles and didactic needs. The findings reveal that students experience conceptual, procedural, and representational obstacles in learning fractions. While parents show strong willingness to support their children's learning, they face limited conceptual understanding, mismatches between school and home learning approaches, and a lack of structured guidance. Based on these findings, an initial didactic design was developed that is contextual, gradual, and adaptive, and

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includes practical guidelines for parents. This design is expected to strengthen learning continuity between school and home and support students' conceptual understanding of fractions.

Keywords: Fractional Learning; Learning Obstacles; Role Of Parents; Didactic Design; Didactical Design Research

INTRODUCTION

Learning fractions is an essential foundation in elementary school mathematics because it serves as a prerequisite for mastering various advanced mathematical concepts, such as rational numbers, proportional reasoning, percentages, statistics, and algebra. A sound understanding of fractions enables students to develop flexible and relational quantitative thinking, which is essential in various mathematical and everyday contexts. Several longitudinal and cognitive studies have shown that a strong understanding of fractions at elementary school age is a significant predictor of long-term mathematics learning success, including achievement in algebra and advanced mathematical reasoning (Artigue, 2020). In the context of modern society, which increasingly relies on numerical literacy, the ability to understand and use fractions meaningfully is also a crucial skill in everyday life, decision-making, and the workplace, which demands precision and quantitative reasoning (Arthur et al., 2020)

Despite their strategic role, fractions are widely recognized as one of the most difficult topics to learn in the school mathematics curriculum. This difficulty is not unique to students in Indonesia but has also been consistently reported in various international studies across educational contexts. Contemporary research indicates that students' difficulties in learning fractions are not solely procedural, but rather primarily lie in their conceptual understanding of fractions as numbers that represent quantitative relationships between parts and wholes (Boonk et al., 2018). Many students struggle with comparing fraction values, determining the order of fractions, and consistently associating fraction symbols with concrete or visual representations.

These difficulties are often related to the phenomenon of *natural number bias* , which is the tendency for students to inappropriately transfer their thinking about whole numbers to fractions. In this situation, students treat the numerator and denominator as two separate numbers without considering the quantitative relationship between them, resulting in persistent and difficult-to-correct misconceptions . Research shows that this bias can emerge early in

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learning fractions and persist into secondary education if not addressed through a conceptually meaningful learning approach (Brousseau, 2020)

Several recent studies also confirm that learning approaches that overemphasize arithmetic procedures, such as memorizing addition or subtraction algorithms for fractions, without reinforcing conceptual meaning, actually result in fragile and unsustainable understanding. Students may be able to solve problems mechanically in the short term, but fail to develop a deep relational understanding of fractions as numbers (Cutting, 2024) This condition causes students to struggle when faced with varied contexts, different representations, or non-routine problems that require conceptual reasoning. Therefore, learning fractions requires designing learning trajectories that allow students to construct meaning gradually through the coordination of concrete experiences, visual representations, and symbolic representations (Cui et al., 2023).

From a didactic perspective, student learning difficulties cannot be understood separately from the interaction between the characteristics of mathematical concepts, students' cognitive readiness, and the learning design implemented by teachers. The Theory of Didactic Situations emphasizes that learning barriers can stem from epistemological, ontogenetic, and didactic aspects (Duval, 2016). Epistemological barriers relate to the nature and complexity of the concept of fractions as non-intuitive rational numbers. Ontogenetic barriers relate to the limitations of elementary school students' cognitive development, which is still highly dependent on concrete experiences in constructing abstract concepts. Meanwhile, didactic barriers arise from the way fraction concepts are presented, sequenced, and represented in learning (Epstein, 2021)

In the context of fraction learning, these barriers are often intertwined. Learning designs that provide insufficient space for exploration of conceptual meaning and coordination of representations can reinforce existing epistemological and ontogenetic barriers. Therefore, a comprehensive analysis of student learning barriers is necessary as a basis for designing responsive and meaningful learning.

On the other hand, elementary school students' mathematics learning takes place not only at school but also at home through parental mentoring. Recent research shows that parental involvement significantly contributes to students' motivation, interest, and mathematics learning outcomes, both directly and through interactions with motivational factors and student characteristics (Fan & Chen, 2021)Supportive, dialogic, and understanding-oriented parental

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involvement has been shown to strengthen students' learning experiences and help overcome learning difficulties that arise at school.

However, several studies also show that the quality of parental involvement is more important than the frequency of involvement alone (Gabriel et al., 2023). In the context of fraction learning, many parents face limited conceptual understanding, differing approaches to learning in schools, and a lack of systematic didactic guidance to support their children's learning at home. Parents tend to use procedural approaches derived from past learning experiences, which do not always align with the conceptual learning approaches developed in schools. This situation has the potential to create a disconnect between learning in school and support practices at home, which can actually reinforce students' misconceptions (Hansen et al., 2017).

Research in Indonesia also shows that parental involvement is positively correlated with elementary school students' mathematics learning outcomes. However, this involvement is generally not explicitly integrated into teachers' instructional design, so parents' role is still positioned as an external supporting factor, not part of the learning system (Hill & Tyson, 2019)As a result, the potential contribution of parents in supporting conceptual learning of fractions has not been optimally utilized.

Although research on fraction learning difficulties and parental involvement has grown, most studies still examine these two aspects separately. Research on fraction learning tends to focus on teacher-student interactions in the classroom, while research on parental involvement often positions parents as external variables influencing learning outcomes. Research that explicitly integrates analysis of student learning barriers, teacher didactic practices, and parental support experiences into a systematic learning design remains relatively limited, particularly in the context of elementary schools in Indonesia.

Based on this gap, this study offers a different approach by integrating the analysis of student learning barriers, teacher learning practices, and the role of parents within a Didactic Design Research (DDR) framework. DDR enables a systematic link between theoretical and empirical analysis of learning barriers and the design of learning designs that are responsive to the real context of learning (Pienaar, 2014)Through the *prospective analysis stage* in DDR, this study aims to explore conceptual barriers to fraction learning as well as the experiences and needs of parents in assisting children learning at home, then formulating them into a hypothetical learning trajectory *and* an initial didactic design that supports the continuity of learning between school and home (Rittle-Johnson et al., 2015).

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This approach is expected to provide theoretical and practical contributions to the development of more conceptual, collaborative, and sustainable fraction learning in elementary schools. Theoretically, this research enriches the study of fraction learning by integrating didactic perspectives and the role of parents within a single analytical framework. Practically, this research is expected to produce an initial didactic design and a contextual and applicable learning support guide, which can be utilized by teachers and parents to support elementary school students' fraction learning in a more meaningful and sustainable manner.

RESEARCH METHODOLOGY

Research Approach and Design

This research uses a qualitative approach within the *Didactic Design Research* (DDR) framework. This qualitative approach was chosen because it allows researchers to deeply understand the experiences, perceptions, and contexts of fraction learning among students, teachers, and parents. The focus of this research is not on quantitatively measuring learning outcomes, but rather on identifying learning obstacles *and* learning needs that emerge in real-world practice (Sariamas et al., 2023).

The DDR framework was used because it systematically integrates theoretical and empirical analysis in mathematics learning design. In this study, DDR was applied with an emphasis on the prospective analysis stage, namely the initial stage aimed at identifying potential student learning barriers and formulating an initial didactic design for fraction learning that supports continuity of learning between school and home, particularly through parental involvement (Siegler et al., 2023).

Research Context and Participants

The research was conducted at a public elementary school in Sukabumi Regency, West Java. This context was chosen because it reflects the typical fraction learning environment found in elementary schools and also demonstrates the importance of parental support in supporting students' learning at home.

The study participants consisted of two elementary school teachers and ten parents of fourth-grade students. Participants were selected using *purposive sampling* , considering their direct involvement in fractions learning. Teachers served as primary informants regarding classroom learning practices, obstacles experienced by students, and didactic limitations

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encountered. Meanwhile, parents provided perspectives on their experiences, challenges, and strategies for supporting fractions learning at home (Siegler et al., 2021).

Data collection technique

Data collection was conducted through in-depth semi-structured interviews and document analysis. Semi-structured interviews were used to obtain rich and flexible data while remaining focused on the research focus. Interviews with teachers covered fraction learning strategies, students' conceptual difficulties, time constraints and learning media, and teachers' perspectives on the role of parents in supporting learning at home (Siegler & Lortie-Forgues, 2017).

Interviews with parents focused on their experiences assisting their children in learning fractions, their understanding of fraction concepts, the differences in approaches between learning at school and at home, and the need for systematic guidance (Suryadi et al., 2019). Additionally, document analysis was conducted on mathematics textbooks, learning tools, student worksheets, and learning modules used in schools, as well as several international references related to fraction learning. This analysis aimed to examine the learning flow, fraction representation, and learning materials supporting parental involvement (Sumantri et al., 2025).

Data Analysis Techniques

Data analysis was conducted qualitatively, referring to Miles and Huberman's interactive analysis model, which includes data reduction, data presentation, and conclusion drawing. Interview data were transcribed and analyzed through open, axial, and selective coding processes to identify key themes related to student learning barriers, didactic constraints faced by teachers and parents, and learning support needs.

Within the DDR framework, the analysis focused on identifying three types of learning barriers: epistemological, ontogenetic, and didactic. Findings from interviews and document analysis were then integrated to develop an initial *Hypothetical Learning Trajectory* (HLT), which served as the basis for designing the didactic design for fraction learning and developing learning support guidelines for parents (Wang et al., 2020).

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Data Validity and Research Ethics

Data validity was ensured through source triangulation, comparing data obtained from teachers, parents, and learning documents. *Member checking* was conducted by confirming interpretations with participants to ensure consistency of meaning. Furthermore, discussions with supervisors and academic colleagues were conducted to enhance the accuracy of the analysis (Wang & Sheikh-Khalil, 2014).

This research was conducted in accordance with ethical research principles. All participants provided informed consent prior to data collection. Participants' identities were kept confidential, and the data collected was used solely for academic purposes. Participants were also given the right to withdraw from the study at any time without any consequences (Wulandari & Bektiningsih, 2023).

RESULTS AND DISCUSSION

Results

The research results are presented based on thematic analysis of interview data with teachers and parents of fourth-grade elementary school students, as well as a review of the context of fraction learning that takes place in the classroom and at home. The presentation of the results focuses on: (1) students' conceptual obstacles in understanding the initial concept of fractions, (2) parents' difficulties in accompanying fraction learning at home, (3) teachers' didactic obstacles in fraction learning, and (4) the need for learning design as a basis for developing initial didactic designs within the *Didactic Design Research* (DDR) framework that supports collaboration between school and home (Zulparis et al., 2021).

Students' Conceptual Obstacles in the Initial Concept of Fractions

The results of the study indicate that fourth-grade students still experience conceptual barriers in understanding fractions as numbers. This barrier is reflected in their tendency to understand fractions procedurally without fully understanding the concept of parts of a whole. Most students interpret fractions in a limited way as visual forms or cut-off images, without understanding the relationship between the numerator and denominator as a quantitative representation of a value.

These conceptual difficulties emerge when students are asked to compare two fractions with different denominators or explain the meaning of a fraction outside the context of a routine

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problem. Teachers revealed that many students are able to perform mechanical exercises but struggle when problems are presented in a variety of contexts or require conceptual reasoning. This indicates that a stable and in-depth understanding of the concept of fractions has not yet developed (Röllke & Großmann, 2022).

Mapping of types of student learning barriers based on analysis within the DDR framework is presented in Table 1 to provide a systematic overview of the character of conceptual barriers that arise in fraction learning.

Table 1. Mapping of Student Learning Barriers to Fraction Material (Based on DDR – Brousseau)

Types of Obstacles	Obstacle Description	Empirical Indications from Data
Epistemological	Students do not yet understand the meaning of fractions as part of a whole	Students think of fractions as just two numbers separated by a line.
	Error in comparing fractions	Students judge that fractions with larger denominators are always of greater value.
Ontogenetic	Students' cognitive readiness is inadequate	Students have difficulty relating the concept of fractions to everyday experiences.
Didactic	Learning emphasizes too much on procedures	The teacher conveys steps without exploring the meaning of the concept.
	Lack of understanding-based, step-by-step training	Students quickly forget and are unable to transfer concepts to different questions.

This obstacle mapping shows that student learning difficulties do not stem solely from individual abilities, but are also influenced by the characteristics of mathematical knowledge (epistemology), student developmental readiness (ontogenetics), and didactic decisions in learning (didactics). These findings provide an important basis for designing didactic anticipations at the learning design stage.

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It should be emphasized that the issues arising from the use of images, symbols, or problem context are not positioned as separate representational barriers, but rather as part of a didactic strategy that is not yet optimal in supporting the development of students' conceptual understanding. Therefore, the focus of the analysis remains directed at students' conceptual barriers in fully understanding the meaning of fractions.

Parents' Difficulties in Accompanying Fraction Learning

Research findings indicate that parents are highly motivated to support their children in learning fractions at home. However, this support still faces various challenges, particularly related to limited understanding of fraction concepts and differences in learning approaches between school and home (Georgiou & Kyza, 2018).

Some parents expressed a tendency to use rote learning methods or mechanical procedures they recall from previous learning experiences. In practice, the assistance provided often involves direct demonstration of problem-solving steps without conceptual explanation. This type of support has the potential to reinforce procedural learning and does not support students' in-depth conceptual understanding (Ikhlas et al., 2021).

A summary of the coding results of teacher and parent interviews regarding obstacles to assisting with fraction learning is presented in Table 2.

Table 2. Coding Results of Teacher and Parent Interviews

Main Theme	Subtheme	Data source	Representative Quotes
Conceptual Difficulties	Parents don't understand fractions	Parents (PO3)	“I often get confused explaining fractions, so I usually just give an example answer.”
Mentoring Strategy	Focus on the end result	Parents (PO6)	"The important thing is that the child can do their homework, whether they understand it or not will be discussed later."

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Main Theme	Subtheme	Data source	Representative Quotes
Media Limitations	There are no props at home	Parents (PO8)	"There are no tools at home, so I just use books."
Teacher Perception	Mentoring is not yet aligned	Teacher A (GA)	“Sometimes the way parents do things is different from the way they do things in class, which can confuse children.”

This table shows that the challenges to parental guidance are not only technical but also didactic. Differences in understanding and approaches between school and home can potentially lead to student confusion if not addressed through structured learning guides aligned with classroom instruction.

Didactic Constraints of Teachers in Learning Fractions

Teachers identified that learning fractions requires more time and strategies than other materials. However, limited learning time and curriculum requirements make it difficult for teachers to provide optimal conceptual understanding. Furthermore, teachers face limitations in providing media and teaching materials that can bridge student understanding while supporting parental involvement (Damopolii et al., 2021). Available teaching materials are generally designed for classroom activities and are not specifically intended for collaborative use between school and home. "In class, I can explain things slowly, but at home, parents don't have the same understanding," (GA). This situation indicates that didactic constraints are not only related to classroom learning practices, but also to limitations in learning design that do not accommodate continuity between learning in school and support at home (Pratiwi & Wuryandani, 2020).

The Need for Learning Design that Supports Collaboration between School and Home

Based on a synthesis of findings from students, teachers, and parents, this study identified the need for a fraction learning design that is gradual, contextual, and easily understood by both students and parents. The learning design is not only intended for classroom learning but also serves as a guide for learning support at home. This need led to the development of an initial didactic design equipped with activities based on everyday contexts, the use of concrete and symbolic representations as didactic strategies, and communicative guidance for parents. This

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design is positioned as a bridge between school and home learning continuity (Ruhma, S. Z., Susilawati, W., & Nuraida, n.d.).

A summary of the learning needs of students and parents along with their didactic implications is presented in Table 3, which forms the basis for designing the initial didactic design and the hypothetical learning trajectory (HLT) in the next stage.

Table 3. Learning Design Needs in Fraction Learning

Aspect	Description of Needs	Didactic Implications
Student Needs	Gradual and meaningful learning	Systematic development of learning paths (HLT)
	Strengthening the basic concept of fractions	Context-based activities and real-life experiences
Parents' Needs	A simple and practical guide	Mentoring module with non-technical language
	Alignment with teacher methods	Didactic design that can be applied at home
Teacher Needs	Anticipating learning barriers	Designing didactic situations based on DDR

The needs shown in Table 3 illustrate the learning design. These needs indicate that fraction learning requires a collaborative approach between school and home. Therefore, the developed didactic design is expected to support students' conceptual understanding sustainably through the synergistic role of teachers and parents.

DISCUSSION

Conceptual Barriers to Fraction Learning from the Perspective of Didactic Theory

In the perspective of *the Theory of Didactic Situations* (Brousseau, 1997), conceptual obstacles experienced by students can be categorized into three main types: epistemological, ontogenetic, and didactic.

Epistemological barriers relate to the intrinsic nature of the concept of fractions as mathematical objects. Fractions are not simply symbols or numbers, but rather quantitative

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representations of the relationship of parts to a whole. Research findings indicate that some students understand fractions only as symbolic or pictorial forms, for example, viewing them $\frac{1}{2}$as semicircles without realizing the numerical relationship between the numerator and denominator. This condition aligns with Brousseau's argument that mathematical understanding limited to representational forms without internalizing abstract meanings will lead to conceptual errors. For example, when asked to compare fractions $\frac{1}{3}$and $\frac{1}{4}$fractions using images, some students choose the visually "bigger" fraction without considering its rational value.

Ontogenetic barriers arise from students' cognitive development, which is not yet fully mature enough to understand the abstract concept of fractions. The Piagetian framework and related research indicate that elementary school children, especially those in grades 4–6, are still in the transition stage from *concrete operational* to *formal operational* . Field findings show that students need concrete experiences such as cutting a cake, dividing paper, or dividing everyday objects to understand the part-whole relationship. Without these concrete experiences, fraction understanding remains partial and procedural. For example, some students can add $\frac{1}{2}+\frac{1}{3}$using the general procedure (equalizing the denominators), but cannot logically explain why the denominators must be equal.

Didactic barriers were the most dominant, namely the limitations of learning designs that did not provide enough space for conceptual exploration. Research findings showed that most classroom activities still emphasized memorizing procedures or working on repetitive problems, so students lacked the opportunity to build relational understanding between fractions, integers, and real-world contexts. For example, when teachers gave problems comparing fractions, most students guessed the answers or used simple manipulative procedures without understanding why those procedures were correct. The dominance of these didactic barriers confirms that understanding fractions is not simply a cognitive issue for students, but also the result of learning designs that are less than optimal in facilitating conceptual meaning (Sari et al., n.d.).

Thus, visual, symbolic, and contextual representations cannot be seen as mere barriers, but rather as didactic tools that must be systematically designed. Implementing strategies such as concrete manipulatives, visualizing fractions in everyday contexts, and reflective discussions are key to overcoming these didactic barriers.

Comparison with Siegler & Lamon's Theory

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The findings of this study reinforce the views of Siegler et al. (Festiawan et al., 2021) that understanding fractions is not simply an expansion of whole numbers, but rather a conceptual reconstruction that requires changes in students' thinking.

Students often have difficulty moving from whole numbers to fractions because whole numbers are understood as whole, linear entities, while fractions require an understanding of the relationship between parts and wholes. Research shows that although students can perform operations with fractions (e.g., addition with unlike denominators), they often fail to predict the outcome or explain the reasoning behind the procedure. This indicates that procedural learning appears successful but is conceptually fragile, consistent with Siegler & Pyke's (2013) findings about unstable procedural knowledge.

The procedural learning implemented by some parents or teachers focuses students on how to solve problems without deepening the meaning of fractions. For example, children are able to solve problems $\frac{1}{2}$ by $\frac{1}{3}$ dequating the denominators, but when faced with contextual situations like dividing pizza or chocolate, they are unable to explain the results. This finding aligns with international research showing that procedural learning that is not linked to concrete representations tends to result in shallow understanding (Damopolii et al., 2021).

Fieldwork findings indicate a similar phenomenon to international studies: conceptual barriers are more dominant than cognitive or motivational barriers. For example, Siegler et al. (Luo et al., 2020) found that children in the United States' fraction understanding required an explicit learning trajectory from concrete experiences to abstraction, similar to the findings in this study. The main difference lies in the context of parental involvement; in Indonesia, parental involvement tends to focus on procedural reinforcement, whereas in some international studies, home learning programs are more directed at strengthening conceptual meaning.

Parental Involvement as a Didactic Issue

The research findings indicate that parents play a significant role in supporting fraction learning at home, but this role does not always support students' conceptual understanding. Within the DDR framework, parents can be positioned as co-didactic agents, not simply supervisors or homework assigners.

Most parents still use a procedural approach, teaching children how to solve problems rather than understanding the meaning of fractions. This disconnect results in students receiving different strategies at home and at school, leading to confusion and reinforcing conceptual

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barriers. For example, students who are taught to add fractions by memorizing steps at home but are asked to explain the part-to-whole relationship in school often fail to bridge the two approaches.

By positioning parents as co-didactic agents, learning design needs to provide concrete guidance for parents to support fraction learning in a way that aligns with HLT in the classroom. This guidance can take the form of examples from everyday activities, household manipulatives, or reflective questions that encourage children to think about the 'why' rather than just the 'how'. For example, parents can help children divide fruit or cookies and ask: "If we eat 2 out of 6 pieces, how many are left? Why is this called a fraction $\frac{2}{6}$?"

The findings of this study confirm that fractions learning design needs to consciously address the continuity between school learning and home learning support. Within this framework, parents are no longer positioned merely as informal facilitators, but rather as part of a didactic system explicitly designed to support students' conceptual learning trajectories. The proposed learning design integrates teaching materials written in language accessible to parents, enabling alignment of conceptual understanding between school and home. Furthermore, home learning activities are designed to align with classroom learning objectives and approaches, ensuring students receive a consistent and meaningful learning experience (Safitri et al., 2023). This design also incorporates reflective strategies and guiding questions that encourage students and parents to discuss the meaning of fraction concepts, rather than simply completing calculation procedures. Thus, didactic continuity between school and home becomes an important foundation for building a more robust and sustainable conceptual understanding of fractions in elementary school students. Thus, parental involvement is no longer ad hoc but becomes part of a learning design that is responsive to the context of students, teachers, and the home environment.

Implications of Findings for Didactic Design in the DDR Framework

Within the framework of *Didactic Design Research* (Simbolon & Koeswanti, 2020), analysis of students' conceptual barriers serves as the primary foundation for developing *a Hypothetical Learning Trajectory* (HLT) and initial didactic design. HLT is designed as a learning trajectory that depicts a sequence of learning experiences that hypothetically enable students to gradually build a conceptual understanding of fractions, from concrete experiences to understanding the meaning of fractions as numbers.

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Based on the synthesis of research findings, the formulated HLT emphasizes contextual fraction learning and is oriented towards strengthening conceptual meaning, utilizing concrete, visual, and symbolic representations as supporting didactic strategies. Representations are used in a gradual and directed manner to help students coordinate various perspectives on the concept of fractions, rather than as the ultimate goal of learning.

Based on the results of the analysis of learning barriers and learning needs, this study formulated a hypothetical learning trajectory (HLT) for the initial learning of fractions, presented in Figure 1, which describes a hypothetical learning trajectory that starts from students' concrete experiences, continues with exploration of the meaning of parts of a whole, to strengthening the concept of fractions as numbers, and is complemented by the role of parental guidance as part of the didactic design.

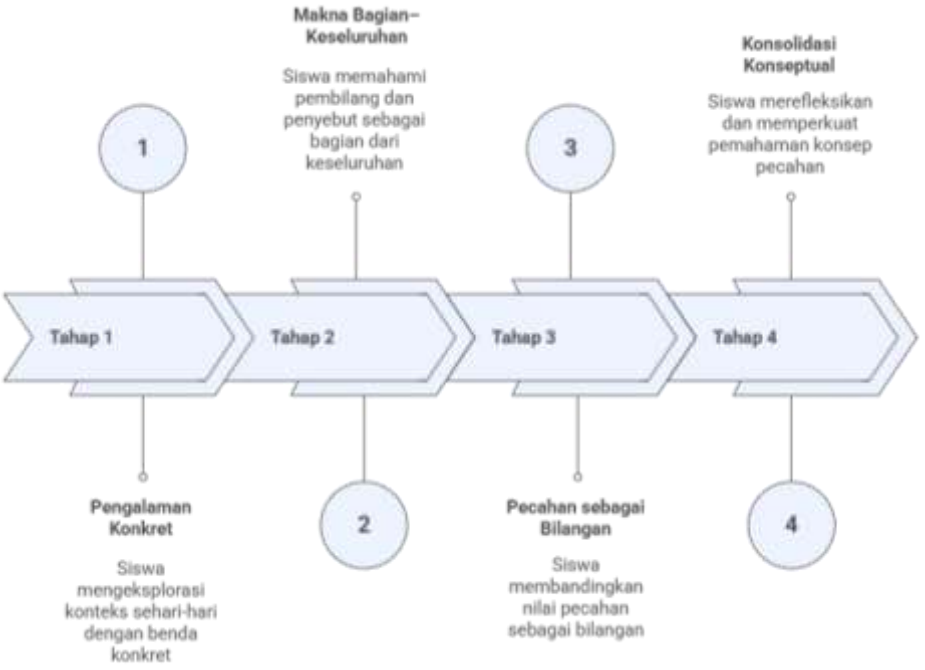


Figure 1. HLT Fraction Learning Based on School and Home Continuity

Furthermore, this didactic design is complemented by the characteristics of accompanying teaching materials that enable parents to act as learning facilitators at home through simple, communicative guidance that aligns with the classroom learning approach. By positioning the

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didactic design as a bridge between school and home learning, this study confirms that didactic continuity is a crucial aspect in overcoming conceptual barriers to fraction learning.

Overall, this discussion confirms that learning fractions in elementary schools requires a learning design that is not only focused on procedural achievement, but is consciously designed to build students' conceptual understanding through collaboration between school and home. The DDR approach allows for a systematic link between theoretical analysis of learning barriers and the design of hypothetical learning trajectories that are responsive to the real contexts of students, teachers, and parents, so that the resulting didactic design has meaningful theoretical and practical contributions.

Theoretical and Practical Implications

The findings of this study have significant theoretical implications for the development of fraction learning studies. From a theoretical perspective, the results confirm that understanding fractions is not simply an extension of whole numbers, but rather a process of conceptual reconstruction that requires students to coordinate various representations: concrete, visual, and symbolic (Ikhlas et al., 2021). The identified epistemological, ontogenetic, and didactic barriers indicate that the concept of fractions has unique characteristics that require a gradual and exploratory learning trajectory. The HLT developed in this study provides a theoretical contribution in the form of empirical evidence that a learning trajectory that begins with concrete experiences, continues with symbolic meaning, and ends with reflection and integration of home-school experiences, can support the formation of a more robust conceptual understanding. Thus, this HLT design enriches the theory of Didactic Design Research (DDR) and offers an integrative model that connects learning barriers with systematic didactic strategies (Georgiou & Kyza, 2018).

From a teacher practice perspective, this research emphasizes the need to shift focus from procedural to conceptual teaching. Teachers serve not only as transmitters of operational steps but also as facilitators who actively guide students in constructing mathematical meaning through concrete exploration, representational coordination, and reflective discussion. Furthermore, parental involvement as co-didactic agents requires teachers to design instructional materials and activities that align between the classroom and home, resulting in more consistent and meaningful student learning experiences. With HLT as a guide, teachers can anticipate student responses, recognize potential conceptual errors, and flexibly adjust didactic interventions. This

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approach not only enhances students' understanding of fractions but also builds critical and reflective thinking skills that can be applied to other mathematical concepts.

In terms of policy and development of elementary school teaching materials, these findings demonstrate the need for structured, contextual learning materials that can be used effectively both at school and at home. Communicative and simple accompanying teaching materials for parents play a crucial role in ensuring continuity of learning and supporting students' conceptual understanding. The development of teaching materials based on the HLT trajectory can set a new standard in developing mathematics learning modules, enabling schools and parents to collaborate synergistically. Education policies that encourage school-home integration and teacher training in the use of HLT and accompanying teaching materials will strengthen the quality of fraction learning nationally.

Reflectively, the integration of theoretical findings, teacher practice, and instructional materials policy confirms that effective fraction learning requires a holistic didactic design. This approach not only fosters students' conceptual understanding but also provides an implementable model that can be replicated in research and practice in elementary school mathematics learning. Thus, the DDR design, which connects learning barriers, learning trajectories, and parental roles, makes a meaningful theoretical and practical contribution to elementary school mathematics education.

Didactic Design Plan and HLT Justification

The didactic design in this study was developed based on the principles of Didactic Design Research (DDR), which places the analysis of learning barriers as the primary foundation in learning design (Esparza et al., 2020). This design is aimed at strengthening the conceptual understanding of fractions as representations of the relationship between parts and wholes through the use of concrete contexts close to students' experiences. The learning trajectory is structured in a gradual and progressive manner, aligned with the cognitive development of elementary school students, thus enabling a meaningful transition from concrete experiences to an abstract understanding of fractions. In addition, the learning design is designed to ensure continuity of the didactic approach between learning at school and learning support at home by involving parents as co-didactic agents. By positioning parents as part of the didactic system, this design is expected to create a consistent and sustainable learning experience, and reduce the

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potential for didactic barriers that arise due to differences in learning approaches in the two environments.

These principles were designed to anticipate epistemological, ontogenetic, and didactic obstacles, while preventing fractional learning that is solely oriented towards procedures. Based on this foundation, an initial hypothetical learning trajectory *(HLT*) was developed as a conceptual framework for linking learning objectives, learning activities, predicted student responses, and anticipated teacher interventions.

Stage 1: Exploration of Concrete Context and Reinforcement of Part–Whole Meaning

The first stage of HLT focuses on exploring everyday contexts relevant to students' experiences, such as sharing cookies, dividing paper, or using everyday objects. The primary goal of this stage is to address epistemological barriers, namely students' difficulty understanding fractions as quantitative representations of parts of a whole.

Justification:

Based on prospective analysis, students tend to view fractions only as symbols or images, without realizing their mathematical meaning. By providing concrete experiences that facilitate direct interaction with real objects, students can build a relational understanding between the numerator, denominator, and the whole. For example, when students are asked to divide 8 cakes into 4 parts, they can directly see the relationship 'parts taken / total parts = fraction'.

Didactic anticipation:

The teacher predicts some student responses, such as focusing on the number of objects taken without considering the whole, or visually assessing parts without counting. The teacher's strategy is to prepare additional reflective and manipulative questions to guide students in seeing quantitative relationships, for example: "If I take 2 out of 4 parts, how many parts are left? Is this result equal to $\frac{1}{2}$?" Thus, the first stage aims to build a conceptual foundation before entering more abstract fraction operations.

Stage 2: Symbolic Meaning and Coordination of Representation

The second stage of HLT emphasizes the symbolic meaning of fractions and the coordination between concrete, visual, and symbolic representations. The focus is on overcoming

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ontogenetic barriers, namely the limitations of students' cognitive development in understanding the abstraction of fractions.

Justification:

Research findings indicate that although students can perform concrete activities, their understanding cannot yet be translated into fraction symbols. This stage integrates concrete manipulatives (e.g., pieces of paper or fraction rods) with symbolic representations ($\frac{1}{2}$, $\frac{1}{3}$) so that students begin to understand that fraction symbols are a way of representing abstract quantities. This process supports the transition from concrete experiences to conceptual reconstruction of rational numbers, in accordance with the findings of Siegler et al. (2011) that understanding fractions requires a change in cognitive structure from integers to rational numbers.

Didactic anticipation:

Teachers prepare for potential student errors, such as comparing fractions solely by their numerators or denominators, or using meaningless, mechanistic procedures. To anticipate this, teachers facilitate fraction comparison activities through dual representation, asking students to visualize the fractions and write them in symbolic form. In this way, the second stage explicitly directs students toward relational and conceptual understanding, rather than merely procedural.

Stage 3: Reflection, Integration, and Parental Involvement

The third stage of HLT emphasizes reflection and integration of knowledge and utilizes parental involvement to respond to didactic barriers, particularly disconnection between school and home.

Justification:

The study results show that most parents still use a procedural approach. Without design interventions, students face inconsistent learning experiences between home and school, potentially reinforcing conceptual barriers. By engaging parents through communicative and contextualized supplementary learning materials, students can continue their learning experiences at home in a structured manner that aligns with the classroom.

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Didactic anticipation:

Teachers anticipate potential student responses, such as confusion due to differences in home and school strategies or dependence on parental assistance. Therefore, accompanying teaching materials include simple yet conceptual activities, reflective questions, and interaction guides that encourage children to explain and justify their answers. For example, when dividing fruit at home, students are asked to write the fraction of each portion and explain their reasoning. This strategy helps students build conceptual consistency while strengthening the role of parents as co-didactic agents.

Integration of HLT Stages and Strengthening of Didactic Design

Overall, the three stages in the Hypothetical Learning Trajectory (HLT) are designed in an integrated manner to strengthen the didactic design of fraction learning. The initial stage focuses on building concrete learning experiences that enable students to understand the concept of fractions as part-whole relationships, thereby minimizing epistemological barriers related to the abstract nature of fractions. Furthermore, the second stage is directed at coordinating various representations, ranging from concrete, visual, to symbolic, so that students are able to construct an understanding of fractions gradually according to their cognitive development and overcome ontogenetic obstacles that arise in the learning process (Röllke & Großmann, 2022). In the final stage, the learning design is expanded to include the role of parents and reflective activities as an effort to respond to didactic obstacles, particularly those related to the discontinuity of learning approaches between school and home. Through the integration of these three stages, this didactic design emphasizes a progressive and flexible learning trajectory, allowing teachers to adjust interventions based on students' actual responses, while simultaneously positioning parents as active participants in the learning process (Machmut et al., 2023). The Didactic Design Research (DDR) approach ensures that HLT serves not only as a theoretical framework but also as a practical tool for designing, observing, and evaluating fractions instruction that is responsive to learning barriers that arise both in the classroom and at home. This design emphasizes a progressive and flexible learning trajectory, allowing teachers to adjust interventions based on student responses, while also making parents an active part of the learning process. The DDR approach ensures that HLT is not only a theoretical framework but also a practical tool for designing, observing, and evaluating fractions instruction that is responsive to real-life learning barriers in the classroom and at home.

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CONCLUSION

This study shows that learning fractions in elementary schools still faces significant conceptual barriers, particularly in understanding fractions as parts of a whole and as numbers. These barriers stem not only from students' cognitive readiness but also from limitations in classroom learning design and the lack of systematic guidance for parents in supporting fraction learning at home. The findings also reveal that parents are highly motivated to participate in their children's learning, but often experience difficulties due to differences in approaches to learning in school and limited conceptual understanding.

Theoretically, this study contributes to the development of fraction learning studies by strengthening the application of *Didactic Design Research* (DDR) as an analytical framework to integrate epistemological, ontogenetic, and didactic barriers in learning design. The findings of this study confirm the relevance of Brousseau's didactic situation theory and hypothetical learning trajectories in understanding the complexity of fraction learning in elementary schools, especially when learning is viewed as a process that occurs across school and home contexts.

From a practical perspective, this research yields an initial didactic design for fractions learning, complemented by a hypothetical learning trajectory and the characteristics of accompanying teaching materials for parents. This design is expected to serve as a reference for teachers in designing more conceptual, contextual, and sustainable fractions learning, while also providing clear guidance for parents in assisting their children in learning at home. Thus, the proposed didactic design has the potential to strengthen school-home collaboration and improve the quality of students' learning experiences.

Further research is recommended to implement and test the effectiveness of the didactic design through subsequent DDR cycles, including retrospective analysis of student responses and parental involvement during the learning process. Furthermore, further research could expand the context and participants and examine the long-term impact of the didactic design on elementary school students' understanding of fractions and attitudes toward learning mathematics.

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